

# POSTERS

6th Iberian Mathematical Meeting



## Posters

**Fri 7th, 19:00** — J.A. Docobo:

*The Mathematical Legacy of Ramón María Aller Ulloa*

**Fri 7th, 19:00** — G. Flores:

*Implementing generating functions to obtain power indices with coalition configuration*

**Fri 7th, 19:00** — T. Igesias:

*Classical genetic algorithms versus genetics algorithms with varying population size over the Wilson functions*

**Fri 7th, 19:00** — J. Sendra: *Symbolic Computation of Drazin Inverses*

6th Iberian Mathematical Meeting  
Santiago de Compostela, October 6th - 8th, 2016

## **THE MATHEMATICAL LEGACY OF RAMÓN MARÍA ALLER ULLOA**

**José Ángel Docobo Durántez<sup>1</sup>**

In observance of the 50th anniversary of the death of Ramón María Aller Ulloa, an illustrious Galician astronomer and mathematician as well as Professor of the University of Santiago de Compostela, this document is to serve as a record of his contributions to the field of Mathematics. Without a doubt, his principal work was the book, ALGORITMIA.

However, it is also important to keep in mind his articles concerning the Parabolic Theory of Errors as well as Sets and Finite, Undefined, and Transfinite Numbers. He also wrote about some of the principals of the Theory of Sets.

**Keywords:** History, Mathematics

**MSC 2010:** 01

### **Referencias**

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## Implementing generating functions to obtain power indices with coalition configuration

J.Rodríguez-Veiga<sup>1</sup>, G.I. Novoa-Flores<sup>2</sup>, B. Casas-Méndez<sup>3</sup>

This work is part of cooperative game theory. Let us consider the class of so-called cooperative games with transferable utility (TU-games, for short), in which agents have the opportunity to distribute the profit generated by cooperation among them. A mechanism to propose asuch distribution of the profits among the agents is called a solution or value. There is a special class of TU-games, weighted majority games, with an important role in social sciences since they can represent a Parliament or any institution where decisions are taken by vote. A solution for weighted majority games is called power index. Since the calculation of power indices can be hard in the presence of a large number of agents, different calculation methods have been proposed. Among these methods, in this setup we will use the so-called generating functions (cf. Brams and Affuso, 1976 and Lucas, 1983 among others). This is based in a technique of combinatorial analysis that allows enumerate the set of possible coalitions among agents, while having control about their respective weights, which it is essential for the final determination of the most important power indices. Specifically, we consider the Banzhaf–Coleman (Albizuri and Aurrekoetxea, 2006) and Owen (Albizuri, Aurrekoetxea, and Zarzuelo, 2006) power indices for weighted majority games modified by a coalition configuration modeling constraints on cooperation among agents. We present calculation algorithms of them that make use of the method of generating functions. We programmed the procedure in the open language R and it is illustrated by a real life example taken from social sciences. To finish, we generalize the algorithms to a wider class of games.

**Keywords:** Weighted majority games, Power indices, Banzhaf–Coleman index, Shapley–Shubik index, Generating functions, Coalition configuration, Weighted multiple majority games

**MSC 2010:** 05A10, 91A12

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## Classical genetic algorithms versus genetics algorithms with varying population size over the Wilson functions

María Teresa Iglesias Otero<sup>1</sup>, Iago García Outeiro<sup>2</sup>

Genetic algorithms (GA) are a mathematical tool inspired upon the mechanisms of natural evolution and are mainly applied in the framework of function optimization. Several measures have been developed for predicting the behavior of the genetic algorithms (order,  $k$ -epistasis, ...) and several classes of “laboratory functions” have been especially designed to study their dynamics. The Wilson functions are of interest to have a good characterization of functions which are easy to optimize by genetic algorithms, particularly if the functions cause difficulties for standard optimization methods. Moreover, the size of the population is one of the most important choices that any user of Genetic Algorithms faces. This parameter have been investigated from different perspectives. Most of them consider Classical Genetic Algorithms (CGA); i.e. GA using binary codification with fixed size of population and classical genetic operators: selection, crossover and mutation. In this note we analyze CGA versus Genetic Algorithms with Varying Population Size (GAVPS) for the Wilson functions. GAVPS do not use any selection mechanism and replace it by the concept of “age” of a chromosome. By means of some experimental results, we show how the  $k$ -epistasis of Wilson functions is strongly related to their behavior.

**Keywords:** genetic algorithm,  $k$ -epistasis, Wilson functions

**MSC 2010:** 68R99

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## Symbolic Computation of Drazin Inverses<sup>\*</sup>

J. Sendra<sup>1</sup>

Many authors have analyzed the properties of Drazin inverses as well as their applications. An important alternative issue in the topic is the computation of the Drazin inverse.

The problem has been approached mainly for matrices with complex numbers. Nevertheless, in a second stage, different authors have addressed the problem of computing Drazin inverses of matrices over other coefficients domains as rational function fields. Furthermore, symbolic techniques have proven to be a suitable tools for this goal. In this context, we show how to compute the Drazin inverses of matrices whose entries are elements of a finite transcendental field extension of a computable field. For this purpose, we reduce the computation of Drazin inverses over certain computable fields to the computation of Drazin inverses of matrices with rational functions as entries. As a consequence, we derive a symbolic algorithm. The algorithm is applied to matrices over the field of meromorphic functions, in several complex variables, on a connected domain and to matrices over the field of Laurent formal power series. Essentially, this algorithmic method applies symbolic computation to determine the Drazin inverse via specializations, and reduces the problem to the computation, via Gröbner bases, of Drazin inverse matrices with multivariate rational functions as entries. Furthermore, we show how to relate the specialization of the Drazin inverse of a matrix, with meromorphic function entries, and the Drazin inverse of the specialization.

More precisely, given a matrix  $A$ , the idea consists in the following three steps: (1) [Specialization step] first we associate to  $A$  a matrix  $A^*$  whose entries are rational functions in several variables, whose entries are rational functions in several variables; (2) [Inverse computation step] we compute the Drazin inverse of  $A^*$ ; (3) [Evaluation step] finally, from the Drazin inverse of  $A^*$ , we get the Drazin inverse of  $A$ .

The results mentioned above have been developed in the papers [1, 2]

**Keywords:** Drazin inverse, Gröbner bases, symbolic computation, meromorphic functions

**MSC 2010:** 15A09, 68W30

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