SPECIAL SESIONS

6th Iberian Mathematical Meeting

S1
Algebra and Combinatorics

Thu 6th, 17:00 - 17:25, Aula 7 – S. A. Lopes:
Hochschild cohomology of rings of differential operators in one variable

Thu 6th, 18:00 - 18:25, Aula 7 – C. D'Andrea:
Ill-posed points for the Rational Interpolation Problem

Thu 6th, 18:30 - 18:55, Aula 7 – A. Cain:
Combinatorial and computational properties of the sylvester monoid

Thu 6th, 19:00 - 19:25, Aula 5 – R. Duarte:
The number of parking functions with center of a given length

Thu 6th, 19:00 - 19:25, Aula 7 – C. Gamas:
Spherical functions and latin squares

Thu 6th, 19:30 - 19:55, Aula 5 – X. García Martínez:
The non-abelian tensor product of different structures

Thu 6th, 19:30 - 19:55, Aula 7 – D. de la Concepción:
On the classification of nilpotent quadratic Lie algebras

Fri 7th, 11:30 - 11:55, Aula 7 – M. Rosas:
On the growth of the Kronecker coefficients

Fri 7th, 12:00 - 19:25, Aula 7 – J. Araújo:
Combinatorics, Number Theory and Groups: the other Name of Semigroups

Fri 7th, 12:30 - 12:55, Aula 7 – A. Fernández:
Jordan techniques in Lie theory

Fri 7th, 13:00 - 13:25, Aula 5 – V. Pérez-Calabuig:
Reduction theorems on generalised kernels of finite

Fri 7th, 13:00 - 13:25, Aula 7 – Y. Cabrera:
Evolution algebras of arbitrary dimension and their decompositions

Fri 7th, 16:30 - 16:55, Aula 7 – F. Botana:
Automated proof and discovery in dynamic geometry

Fri 7th, 17:30 - 17:55, Aula 7 – E. Sáenz de Cabezón:
Combinatorial computer algebra for network analysis and percolation

Fri 7th, 18:00 - 18:25, Aula 7 – R. Mamede:
Gray codes for noncrossing and nonnesting partitions of classical types

Fri 7th, 18:30 - 18:55, Aula 7 – N. Rego:
Universal alpha-central extensions of Hom-Leibniz n-algebras

Sat 8th, 9:30 - 9:55, Aula 7 – A. J. Breda d'Azevedo:
Regular pseudo-oriented maps

Sat 8th, 10:00 - 10:25, Aula 7 – M. A. Marco Buzunariz:
Combinatorial conditions for linear systems of projective hypersurfaces

Sat 8th, 10:30 - 10:55, Aula 7 – J. C. Costa:
On omega-identities over finite aperiodic semigroups with commuting idempotents

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Hochschild cohomology of rings of differential operators in one variable

Samuel A. Lopes[1]

A polynomial h in the variable x determines the derivation $h\frac{d}{dx}$ of the polynomial ring $\mathbb{F}[x]$ and, together with the multiplication by x operator, it generates a noncommutative algebra A_h whose elements can be written as differential operators with coefficients in $\mathbb{F}[x]$. I will talk about some features of this algebra related to derivations and the structure of the Hochschild cohomology Lie algebra of A_h, both in prime and zero characteristics. I will then explain how the complete Hochschild cohomology can be determined and what this says about the formal deformations of A_h.

This is joint work with G. Benkart and M. Ondrus, and work in progress with A. Solotar.

Keywords: Hochschild cohomology, Weyl algebra, Witt algebra, derivations, Ore extensions

MSC 2010: 16S32, 16W25

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Ill-posed points for the Rational Interpolation Problem

Teresa Cortadellas Benítez[1], **Carlos D'Andrea**[2] **Eulàlia Montoro**[2]

For pairs of points $(u_1, v_1), \ldots, (u_n, v_n)$ with coordinates in a field K, with $u_i \neq u_j$, and $1 \leq k \leq n$, if $i \neq j$, the Rational Interpolation Problem consists in deciding whether there exist (and if so, compute) polynomials $N_{k-1}(x)$, $D_{n-k}(x) \in K[x]$ of degree bounded by $k-1$ and $n-k$ respectively such that $D_{n-k}(u_i) \neq 0$ for all $i = 1, \ldots, n$ and

$$\frac{N_{k-1}(u_i)}{D_{n-k}(u_i)} = v_i,\ i = 1, \ldots, n. \tag{1}$$

When $k = n$, this problem reduces to the well-known *Lagrange Interpolation* problem. In contrast with this classical problem, there is not always a solution for the Rational Interpolation Problem for any given input data. In this talk, we will present the problem, show some algebraic formulations of it, and a geometric description of the set of ill-posed points (those for which there is no solution to (1)).

Keywords: Rational Interpolation, Vandermonde Matrices, Weak Rational Interpolation, Minimal Solutions, Filtrations

MSC 2010: 13P15, 15A15, 68W30

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Combinatorial and computational properties of the sylvester monoid

Alan J. Cain[1], **Robert Gray**[2], **António Malheiro**[1]

The sylvester monoid is the monoid of right strict binary search trees. It was introduced to give a new construction of the Loday–Ronco hopf algebra, and is one of a family of 'plactic-like' monoids whose elements can be identified with combinatorial objects [2].

This talk will discuss recent work on the sylvester monoid from two perspectives: computing and combinatorics. On the computational side, it will discuss how the sylvester monoid is automatic (in the sense of the automatic groups of Epstein et al. [1]) and is presented by a convergent rewriting system. On the combinatorial side, it will discuss the 'cyclic shift' relation, which relates elements that factor as xy and as yx. If one builds a graph whose vertices are the elements of the sylvester monoid, and whose edges are given by the cyclic shift relation, then the connected components of this graph are finite and have a very neat characterization, and the connected components have bounded diameter (although the number of vertices in a component is unbounded).

Keywords: sylvester monoid, binary search trees, automatic, cyclic shifts

MSC 2010: 20M35, 05C12

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The number of parking functions with center of a given length

Rui Duarte[1], **António Guedes de Oliveira**[2]

Let $1 \leq r \leq n$ and suppose that, when the *Depth-first Search Algorithm* is applied to a given rooted labelled tree on $n + 1$ vertices, exactly r vertices are visited before backtracking. Let R be the set of trees with this property. We count the number of elements of R.

For this purpose, we first consider a bijection [3], due to Perkinson, Yang, and Yu, that maps R onto the set of parking function with *center* [1] of size r. A second bijection maps this set onto the set of parking functions with *run* r. We then prove that the number of length n parking functions with a given run is the number of length n *rook words* [2] with the same run. This is done by counting related lattice paths in a ladder-shaped region. We finally count the number of length n *rook words* with run r, which is the answer to our initial question.

Keywords: parking functions, bijections

MSC 2010: 05A15, 05A19, 05C30

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Spherical functions and Latin squares

Carlos Gamas[1]

The Alon-Tarsi Conjecture states that for even n, the number of even latin squares of order n differs from the number of odd latin squares. In this note we prove that this conjecture is true if and only if there exists a permutation $\zeta \in S_{n^n}$ and a spherical function, φ, such that $\varphi(\zeta) \neq 0$.

Keywords: spherical functions, latin squares

MSC 2010:

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The non-abelian tensor product of different structures

Xabier García-Martínez[1]

The non-abelian tensor product of Lie algebras was introduced by Ellis in [1]. In this talk we are going to introduce the different generalizations of this object to several structures as Lie superalgebras, Leibniz algebras and superalgebras and Lie-Rinehart algebras. The non-abelian tensor product is a very important tool that relates the Lie algebra homology with cyclic homology, non-abelian homology or central extensions. We are going to use this object to have simpler proofs of known theorems as well to obtain new results.

Keywords: Non-abelian tensor and exterior products, Lie superalgebras, homology

MSC 2010: 17B55, 17B30, 17B60

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On the classification of nilpotent quadratic Lie algebras

Daniel de-la-Concepción[1], **Pilar Benito**[1] **Jesús Laliena**[1]

A key fact about simple (semi-simple) Lie algebras is that an invariant non-degenerate bilinear form, the Killing form, can be defined on them. The existence of such a form is a main ingredient for the physical theories associtated to them.

Nevertheless, there are more Lie algebras with such a nice bilinear form. Such a structure is called a quadratic Lie algebra and it is a pair (L, B) where L is a Lie algebra and $B : L \times L \to L$ is a non-degenerate bilinear form invariant in the sense that $\forall x, y, z \in L,\ B([x, y], z) = B(x, [y, z])$.

In this talk I will concentrate in the nilpotent Lie algebras and I will show the complexity of classifiying such algebras by stating an equivalence of categories that relates them to orbits of the group of automorphism of the free Lie algebra acting on a specific Lie module.

Keywords: Quadratic Lie algebra, Invariant Bilinear form, Equivalence of categories

MSC 2010: 17B30, 15A63

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On the growth of the Kronecker coefficients.

Mercedes Helena Rosas[1],

We will introduce the Kronecker coefficients, and explore some of their occurrences in the theory of symmetric functions, and in the representation theory of the general lineal group, and the symmetric group,. Then we will present some of their main known properties. Finally we will report on some recent results on the rate of growth experienced by the Kronecker coefficients as we add cells to the rows and columns indexing partitions. This is join work with Emmanuel Briand and Amarpreet Rattan.

Keywords: Representations theory of the symmetric group, symmetric functions

MSC 2010: 05E10; 05E05

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Combinatorics, Number Theory and Groups: the other Name of Semigroups

João Araújo[1]

During the XIXth and early XXth centuries some remarkable breakthroughs came from the idea, probably first considered by Galois, of extracting information about an object from its *morphisms*. Therefore it came as no surprise that a conviction started spreading: *to every mathematical object there is some semigroup attached, and in many cases that semigroup contains very important information about the original object.* This rough idea was reduced to a mathematical question by Stanislaw Ulam [4] when he proposed the general problem of checking when a natural class of mathematical objects has the property that

$$\mathrm{End}(A) \cong \mathrm{End}(B) \Rightarrow A \cong B\,.$$

In words,*which natural classes of mathematical objects possess the property that the endomorphism monoid of an object contains enough information about the object itself to the point of uniquely identifying it?*

However, this expressive power of semigroups was viewed as being more of a weakness than that of a strength. If nearby every mathematical object is a semigroup encoding critical information about the object itself, then semigroups encode critical information about the whole of mathematics and hence non-trivial statements about them compare to non-trivial statements about *all* of mathematics.

After some preparatory results obtained both in the East and in the West, this depressing view of semigroups was dramatically changed by J.A. Green and D. Rees. Not only important and deep results could be proved about semigroups, but —the good news— their local structure is quite transparent and well described. In the words of John Rhodes *we fully understand the local structure of semigroups*. Therefore the depressing meta-mathematical considerations were replaced by an equally meta-mathematical euphoria:

- nearby every object is a semigroup encoding relevant information about the object;
- many mathematical objects are very important to the world (computer science, physics, etc.);
- therefore semigroups are very important to the world and, despite what was previously thought, they are tractable so that to help the world we have to dive into semigroup theory.

As a consequence, for more than three decades papers in semigroup theory consistently appeared in the most reputable mathematics journals. The background of Green (group theory) and of Rees (ring theory) to a large extent shaped the future work on semigroups; in the words of John Rhodes, semigroups started to be seen as *defective groups* or as *the multiplicative part of a ring*.

To study an object (for example, the natural numbers) we start by identifying a relevant subobject (set of prime numbers) that in some sense controls the behaviour of the whole object. In semigroup theory there are two obvious candidates to be taken as this relevant subset: the group of units (the non-defective part of the defective group) or the idempotents. *Prima facie* all bets should be put on the group of units. It is well known that semigroups are oriented, with parts *above* and parts *below*; it is also known that the parts *below* never generate the parts *above* (rank 1 matrices do not generate rank 2 matrices); the group of units is above everything else; a semigroup S with group of units G is in fact the union $S = \cup_{a\in S\setminus G}\langle\{a\}\cup G\rangle$; and groups are among the most studied of all classes of algebras. On the other hand, idempotents, in general, do not even form an algebra (the product of two idempotent matrices does not need to be idempotent), and they were even less well understood than groups, then…and now! There was only one small detail left: the nature of the questions semigroups pose to groups.

As a sample consider the following ones (where $N := \{1,\dots,n\}$, $\mathcal{S}_n$ denotes the symmetric group on N, and $\mathcal{T}_n$ denotes the full transformation semigroup on N):

1. Let $i \le j \le n$. Classify the groups $G \leqslant \mathcal{S}_n$ such that for every i-set $I \subseteq N$ and for every j-set $J \subseteq N$, there exists $g \in G$ such that $Ig \subseteq J$; of course, when $i = j$ these are just the i-homogeneous groups.

2. Let $k < n$ be two natural numbers. Classify the groups $G \leqslant \mathcal{S}_n$ such that in the orbit of every k-set contained in N there exists a section for every k-partition of N; when $k = 2$ this property is just primitivity; but what happens with the remaining k?

3. Let λ be a partition of n. Classify all the groups $G \leqslant \mathcal{S}_n$ transitive on the type λ partitions of N.

4. Classify the primitive groups $G \leqslant \mathcal{S}_n$ such that there exists a partition P of N and a set $S \subseteq N$ such that Sg is a section for P, for all $g \in G$.

5. Classify the primitive groups with diameter at most n and that do not satisfy the previous property. (The diameter of a finite group is the maximum diameter of its Cayley graphs.)

6. Consider a primitive group $G \leqslant \mathcal{S}_n$ and a transformation $a \in \mathcal{T}_n \setminus \mathcal{S}_n$ such that the kernel type of a is non-uniform. Is it true that $\langle a, G\rangle$ generates a constant map?

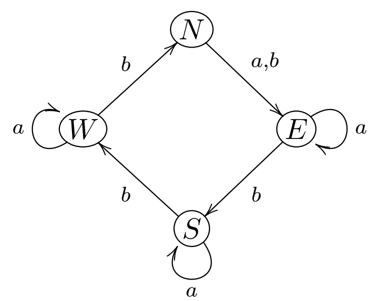
7. Classify the groups $G \leqslant \mathcal{S}_n$ such that for all non-invertible transformations $a \in \mathcal{T}_n \setminus \mathcal{S}_n$ we have

$$\langle \{a\} \cup G \rangle \setminus G = \langle g^{-1}ag \mid a \in G \rangle.$$

Now, for a moment, let's go back to the sixties or early seventies and imagine some semigroupist asks you any of these questions; would you find them anything but a brick wall? Apparently that was what happened and hence semigroupists turned to the study of how idempotents shape the structure of a semigroup. John M. Howie's book can be seen as a brilliant survey of that approach.

The fact that these questions are very challenging does not imply *per se* that these are relevant problems. But maybe they are.

It is well known that ordered pears are sold at an higher price than unordered pears. Therefore, to help farmers moving up the value chain we have to provide them with a robust device that receives pears in any order and delivers all of them oriented. Here, just for the sake of explanation, suppose that when the pears enter in a conveyor belt each one is either pointing to the North, or to the South, or to the West or to the East; of course farmers deal with a much more complex situation, but mathematicians have yet to reach their level. Now it is possible to order the pears just using obstacles of two different kinds along the pears' trip on the conveyor belt, modelled by the following automaton (a step-by-step construction and explanation of this device can be found in [5], pages 24–26):



Observe that if you follow the sequence ab^3ab^3a, you will always end in E. This means that if all the pears are subjected to that sequence of obstacles, they all will end up pointing to the East, irrespective of their original orientation. Such a sequence is called a *reset word*.

One of the oldest conjectures in automata theory, the Černý conjecture (published in 1964), says that a given automaton on n states either does not admit a reset word, or there exists one of length $(n-1)^2$. Observe that the automaton above has 4 states and the given reset word has length $(4-1)^2$; in addition it can be seen that there is none shorter; therefore, the bound $(n-1)^2$, if true, would be optimal. An enormous effort has been made to prove this conjecture, but so far without success. One step towards the solution would be to solve problem (5) above.

However, suppose the Černý conjecture turned out to be true and a farmer approaches your department with a real world (non-simplified) automaton with millions of states and asks if it admits a reset word. It would be extremely vexing if the department replies: *We do not know if that automaton admits a reset word; but if so, then its length is at most* $(n-1)^2$. Therefore, to avoid sounding funny, in addition to (5) mathematicians really need to answer questions (4) and (6) above (please see [1, 2, 3]). However, as previously observed, in 1964 these questions would have looked like a solid brick wall.

The first step towards a change of the tide occurred in 2005; I went to a conference in St. Andrews and in an idle conversation, a group theorist told me that *with the classification of finite simple groups, we can answer any question on finite permutation groups*. Great! Such a bold statement, that probably he would not repeat today, was instrumental for me to unearth some of the questions above: if now they can answer everything, the wall will fall down.

The second step occurred in 2006 in the banquet in honor of a marvelous supervisor, John Fountain. By accident I sat next to Stephen Donkin and after introducing ourselves, I commented *I am glad you are a group theorist because I have a number of questions in group theory.* He just replied *try me!* and I shot question (4) above. He thought for a moment and then said: *that is a very interesting question; you should ask it to Peter M. Neumann in Oxford because he is going to be interested.* And in the end of the meal he reinforced: *please do not forget to write to Peter Neumann; if you have difficulties please write to me.* But there were no difficulties: Peter Neumann replied almost instantaneously! The wall was severely shaken [3].

In this talk I will give a brief survey of what has been going on in this part of mathematics.

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Jordan techniques in Lie theory

Antonio Fernández López[1]

There are no Jordan algebras; there are only Lie algebras (I. L. Kantor). Of course this can be turned around; *nine times out of ten, when you open up a Lie algebra you find a Jordan algebra inside which makes it tick* (K. McCrimmon).

As will be seen in this talk, the dictum of McCrimmon is quite correct. We associate to any ad-nilpotent element of index 3 of a Lie algebra, a Jordan algebra [3] that inherits all good properties of the Lie algebra and reflects the nature of the Jordan element in its structure. Since this transference of properties also works in the opposite direction, we have at our disposal a useful device to deal with Lie problems by means of Jordan theory.

Due to the short duration of the talk, I will only present, as an application of this technique, a proof [2] of the existence of extremal elements in finitary Lie algebras [1]. I will also mention a recent paper of E. Zelmanov [4] that significantly extends the positive solution of the Restricted Burnside Problem, and where the Jordan-Lie connection plays a fundament role.

Keywords: Lie algebras, Jordan algebras, finitary algebras

MSC 2010: 17B65, 17C10

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Reduction theorems on generalised kernels of finite semigroups

Vicente Pérez Calabuig[1],

The problem of computing kernels of finite semigroups goes back to the early seventies and became popular among semigroup theorists through the Rhodes Type II conjecture which proposed an algorithm to compute the kernel of a finite semigroup with respect to the class of all finite groups. Proofs of this conjecture were given in independent and deep works by Ash and Ribes and Zalesskiĭ, and the results of these authors that led to its proof have been extended in several directions. A general treatment of the question is presented for any variety of groups as well as reduction theorems that reduce the problem to simpler structures.

This is joint work with professor Adolfo Ballester-Bolinches.

Keywords: Finite semigroup, Kernels, Inverse semigroup, Varieties

MSC 2010: 20M07, 20M10, 20M17, 20M18

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Evolution algebras of arbitrary dimension and their decompositions

Yolanda Cabrera Casado[1],

We study evolution algebras of arbitrary dimension. We analyze in deep the notions of evolution subalgebras, ideals and non-degeneracy and describe the ideals generated by one element and characterize the simple evolution algebras. We also prove the existence and unicity of a direct sum decomposition into irreducible components for every non-degenerate evolution algebra . When the algebra is degenerate, the uniqueness cannot be assured.

The graph associated to an evolution algebra (relative to a natural basis) will play a fundamental role to describe the structure of the algebra. Concretely, a non-degenerate evolution algebra is irreducible if and only if the graph is connected. Moreover, when the evolution algebra is finite-dimensional, we give a process (called the fragmentation process) to decompose the algebra into irreducible components.

Keywords: Evolution algebra, evolution subalgebra, evolution ideal, non-degenerate evolution algebra, simple evolution algebra, graph associated, reducible evolution algebra, irreducible evolution algebra

MSC 2010: 17D92

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Automated proof and discovery in dynamic geometry*

Francisco Botana[1]

The talk describes the application of algebraic approaches to a geometric interface with millions of users worldwide. More specifically, new abilities concerning automatic proof and discovery in GeoGebra are detailed. A taxonomy for the exact computation of plane algebraic loci and a protocol for computing plane envelopes are recalled. Several methods currently implemented in GeoGebra for general proving are discussed, and our first results dealing with automatic discovery in elementary diagrams are also described.

This is joint work with several colleagues of our MTM2014-54141-P research group and with colleagues from the GeoGebra developers team.

A few, closely related, references are appended in the bibliography.

Keywords: Automatic reasoning in geometry, Interactive learning environments, Dynamic geometry

MSC 2010: 68W30, 68T35

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Combinatorial computer algebra for network analysis and percolation

Fatemeh Mohammaddi[1], **Eduardo Sáenz-de-Cabezón**[2], **Henry Wynn**[3]

The study of networks and other coherent systems using tools from combinatorial commutative algebra has proven to be a fruitful area of research in the last years. The main idea is to associate a monomial ideal to the system under study and obtain its properties (reliability, components' importance, robustness, etc) by studying the algebraic features of the ideal (Hilbert series and function, resolutions, primary decompositions, etc). When applying the results of this approach to medium and large size problems one needs a computer algebra approach. In this respect, the algorithms of commutative computer algebra appear to be efficient for the problems treated.

Keywords: Monomial ideals, Hilbert series, computer algebra, networks, algebraic reliability, percolation

MSC 2010: 13P25, 13D02, 05E40, 60K35

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Gray codes for noncrossing and nonnesting partitions of classical types

Alessandro Conflitti[1], **Ricardo Mamede**[2]

A Gray code is a listing structure for a set of combinatorial objects such that some consistent (usually minimal) change property is maintained throughout adjacent elements in the list. I shall present combinatorial Gray codes and explicit designs of efficient algorithms for lexicographical combinatorial generation of the sets of noncrossing and nonnesting set partitions of length n and types A, B and D. This is a joint work with Alessandro Conflitti.

Keywords: Gray code, Hamilton cycle, Weyl groups, noncrossing partition, nonnesting partition.

MSC 2010: 05A18, 68W99, 94B25.

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Universal α-central extensions of Hom-Leibniz n-algebras*

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Deformations of algebra structures by means of endomorphisms give rise to Hom-algebra structures. They are motivated by discrete and deformed vector fields and differential calculus. Part of the reason to study Hom-algebras is its relation with the q-deformations of the Witt and the Virasoro algebras (see [9]). In this way, deformations of algebras of Lie and Leibniz type were considered, among others, in [5, 9]. The generalizations of n-ary algebra structures, such as Hom-Leibniz n-algebras (or n-ary Hom-Nambu) have been introduced in [1] as triples $(\mathcal{L}, [-,\ldots,-], \widetilde{\alpha})$ consisting of a $\mathbb{K}$-vector space $\mathcal{L}$ equipped with an n-linear map $[-,\ldots,-] : \mathcal{L}^{\times n} \longrightarrow \mathcal{L}$ and a family $\widetilde{\alpha} = (\alpha_i), 1 \leq i \leq n-1$ of linear maps $\alpha_i : \mathcal{L} \longrightarrow \mathcal{L}$, satisfying the following fundamental identity:

$$[[x_1, x_2, \ldots, x_n], \alpha_1(y_1), \alpha_2(y_2), \ldots, \alpha_{n-1}(y_{n-1})] = \\ \sum_{i=1}^{n} [\alpha_1(x_1), \ldots, \alpha_{i-1}(x_{i-1}), [x_i, y_1, y_{2,}, \ldots, y_{n-1}], \alpha_i(x_{i+1}), \ldots, \alpha_{n-1}(x_n)] \tag{1}$$

for all $(x_1, \ldots, x_n) \in \mathcal{L}^{\times n}, y = (y_1, \ldots, y_{n-1}) \in \mathcal{L}^{\times (n-1)}$.

When these twisting maps are all equal to the identity map, then one recovers Leibniz n-algebras [7]. In case $n = 2$, identity (1) is the Hom-Leibniz identity (2.1) in [5], so Hom-Leibniz 2-algebras are exactly Hom-Leibniz algebras.

The goal of this talk is to introduce and characterize universal α-central extensions of Hom-Leibniz n-algebras. In case $n = 2$ we recover the corresponding results on universal α-central extensions of Hom-Leibniz algebras in [5]. Moreover, in case $\alpha = Id$ we recover results on universal central extensions of Leibniz n-algebras in [3] and, in case $n = 2$ and $\alpha = Id$, we recover results on Leibniz algebras from [4].

The talk is organized as follows: we introduce the necessary basic concepts on Hom-Leibniz n-algebras and construct the homology with trivial coefficients of Hom-Leibniz n-algebras. Bearing in mind [2], we endow a Hom-Leibniz n-aslgebra $\mathcal{L}$ with a structure of $(\mathcal{D}_{n-1}(\mathcal{L}) = \mathcal{L}^{\otimes n-1}, \alpha')$-symmetric Hom-co-representation as Hom-Leibniz algebras and define the homology with trivial coefficients of $\mathcal{L}$ as the Hom-Leibniz homology $HL_*^{\alpha}(\mathcal{D}_{n-1}(\mathcal{L}), \mathcal{L})$.

Based on the investigation initiated on [5] and motivated by the fact that the category of Hom-Leibniz n-algebras doesn't satisfy the so called in [8] **UCE** condition, namely the composition of central extensions is central, we generalize the concepts

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of (α)-central extension, universal (α)-central extension and perfection to the framework of Hom-Leibniz n-algebras. We also extend the corresponding characterizations of universal (α)-central extensions. In particular, we show their interplay with the zeroth and first homology with trivial coefficients.

We also introduce the concept of non-abelian tensor product of Hom-Leibniz n-algebras that generalizes the non-abelian tensor product of Hom-Leibniz algebras in [6] and Leibniz n-algebras [3], and we establish its relationship with the universal central extension.

Keywords: Hom-Leibniz n-algebra, universal (α)-central extension, non-abelian tensor product, unicentral Hom-Leibniz n-algebra.

MSC 2010: 17A30, 17B55, 18G60

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Regular pseudo-oriented maps

António Breda d'Azevedo[1]

Despite not being a topological property, pseudo-orientability (introduced by Steve Wilson in the eighties to distinguish some nonorientable maps) has interesting resemblances with orientability when restricted to maps (cellular embeddings of "multiple" graphs on compact connected surfaces). The classification of orientable maps is in a very advanced stage, however this does not happen to pseudo-orientable maps. In this talk I speak about the classification of regular pseudo-oriented maps of small genus and of prime characteristic.

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Combinatorial conditions for linear systems of projective hypersurfaces.

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It is well known that five points in generic position in a projective plane determine one conic. Four points determine an infinite family of such conics, called a pencil. The pencils of curves have been a subject of study since the very beginning of algebraic geometry.

One of the usual problems related with their study is the determinacy of conditions for three or more curves to belong in a pencil. The first important result in this direction was given by M. Noether ([4]) in his famous "$AF + BG$" theorem, that reduced this problem to check local conditions on the base points of the pencil.

In the current century, there have been several improvements to this result that weakened this local conditions to purely combinatorial ones. First for the specific case of line arrangements ([2]) and later for general curves ([1]).

The talk will survey these results and introduce their possible generalizations to higher dimensions; including some known results and open conjectures.

Keywords: Projective varieties, Pencils

MSC 2010: 14N10

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On ω-identities over finite aperiodic semigroups with commuting idempotents

José Carlos Costa[1], **Mário J. J. Branco**[2]

The notion of *pseudovariety* plays a key role in the classification of finite semigroups. Recall that a pseudovariety of semigroups is a class of finite semigroups closed under taking homomorphic images of subsemigroups and finitary direct products. An inverse semigroup is a regular semigroup whose idempotents commute. In [1], Ash proved that the pseudovariety generated by finite inverse semigroups is precisely the pseudovariety $\mathbf{ECom}$ of idempotent commuting semigroups. Let $\mathbf{AInv}$ be the pseudovariety generated by finite aperiodic (i.e., group free) inverse semigroups and let $\mathbf{AECom}$ be the pseudovariety of aperiodic semigroups with commuting idempotents. Surprisingly, Higgins and Margolis [2] have shown that these pseudovarieties are not equal. They did it by exhibiting a certain finite aperiodic semigroup with commuting idempotents and by showing that it does not divide any finite aperiodic inverse semigroup.

In this talk we will present a new proof of the above result by showing that the pseudovarieties $\mathbf{AInv}$ and $\mathbf{AECom}$ do not satisfy the same pseudoidentities. For this we study the word problems for ω-terms (formal expressions obtained from letters of an alphabet using only the operations of multiplication and ω-power) over each of these pseudovarieties. It is shown that both problems are decidable and have different solutions.

MSC 2010: Primary 20M05, 20M07; Secondary 68Q70

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