

# SESIONES ESPECIALES

Congreso RSME 2013



# S1

## Métodos numéricos para la resolución de ecuaciones no lineales

**Jue 24, 11:00 - 11:25, Aula 7** — J. L. Hueso y E. Martínez:

*Semilocal convergence of a family of iterative methods in Banach spaces*

**Jue 24, 11:30 - 11:55, Aula 7** — A. Ezquerro, M. A. Hernández, M. J. Rubio y A. I. Velasco:

*An improvement of the accessibility of Steffensen's method*

**Jue 24, 12:00 - 12:25, Aula 7** — S. Amat, S. Busquier, A. A. Magreñán y N. Romero:

*On a family of two-step relaxed Newton-type methods*

**Jue 24, 12:30 - 12:55, Aula 7** — M. Gutiérrez y A. A. Magreñán:

*Una visión general sobre la convergencia del método de Newton amortiguado*

**Jue 24, 13:00 - 13:25, Aula 7** — S. Amat, S. Busquier y P. Pedregal:

*Aproximación de sistemas de ecuaciones no lineales y no estacionarios*

**Jue 24, 13:30 - 13:55, Aula 7** — V. Candela y J. C. Trillo:

*Sobre unos métodos tipo secante de alto orden de convergencia y libres de derivadas*

**Jue 24, 17:00 - 17:25, Aula 7** — V. F. Candela y R. M. Peris:

*On the Application of Rates of Multiplicity for Finding Singular Roots of Nonlinear Equations*

**Jue 24, 17:30 - 17:55, Aula 7** — R. M. Peris y V. F. Candela:  
*Strategies for solutions of ill conditioned nonlinear equations*

**Jue 24, 18:00 - 18:25, Aula 7** — A. Cordero, J. García, J. R. Torregrosa y M. P. Vassileva:  
*Dinámica de la familia de King de métodos iterativos*

**Jue 24, 18:30 - 18:55, Aula 7** — J. Segura:  
*Evaluación numérica de ceros complejos de funciones especiales*

**Jue 24, 19:00 - 19:25, Aula 7** — S. Amat, M. J. Légaz y P. Pedregal:  
*Aproximación de sistemas Hamiltonianos usando una nueva técnica variacional*

**Jue 24, 19:30 - 19:55, Aula 7** — M. Giusti y J-C. Yakoubsohn:  
*Tracking multiplicities*

## Semilocal convergence of a family of iterative methods in Banach spaces

José L. Hueso<sup>1</sup>, Eulalia Martínez<sup>2</sup>

In [1], the authors introduced a family of third and four order methods for non-linear systems, defined by

$$y_n = x_n - a\Gamma_n F(x_n) \quad (1)$$

$$z_n = x_n - \Gamma_n(F(y_n) + aF(x_n)) \quad (2)$$

$$x_{n+1} = x_n - \Gamma_n(F(z_n) + F(y_n) + aF(x_n)), \quad (3)$$

where  $\Gamma_n = F'(x_n)^{-1}$ . This method is very interesting in terms of efficiency, as it can be seen in the work, because it only uses the first order Frechet-derivative.

In this work, we analyze the semilocal convergence by using recurrence relations, a technique that has been widely studied by Hernandez et al., [2]–[6]. We give a theorem that establishes the existence and uniqueness of the solution, proves the R-order of the method, and finds the a priori error bounds.

Finally, we present some numerical results in order to illustrate the theoretical results.

**Keywords:** Nonlinear system, Iterative method, Banach space, Recurrence relations, semilocal convergence, R-order.

**MSC 2010:** 65J15

## References

- [1] J.L. HUESO, E. MARTÍNEZ, J.R. TORREGROSA, Third and fourth order iterative methods free from second derivative for nonlinear systems, *Applied Mathematics and Computation*, **211**, 190–197 (2009).
- [2] J.M. GUTIÉRREZ, M.A. HERNÁNDEZ, Recurrence Relations for super-Halley Method, *Journal of Computational and Mathematics Applied*, **7**, 1–8 (1998).
- [3] J.A. EZQUERRO, M.A. HERNÁNDEZ, Recurrence Relations for Chebyshev-like Methods, *Applied Mathematics and Optimization*, **41**, 227–236 (2000).
- [4] J.A. EZQUERRO, M.A. HERNÁNDEZ, New iterations of R-order four with reduced computational cost, *BIT Numer. Math.*, **49**, 325–342 (2009).

- [5] J.A. EZQUERRO, M.A. HERNÁNDEZ, D. GONZÁLEZ, Majorizing sequences for Newton's method from initial value problems, *Journal of Computational and Applied Mathematics*, **236** 9, 2246–2258 (2012).
- [6] J.A. EZQUERRO, M.A. HERNÁNDEZ, N. ROMERO, Solving nonlinear integral equations of Fredholm type with high order iterative methods, *Journal of Computational and Applied Mathematics*, **236** 6, 1449–1463 (2011).

<sup>1</sup>Instituto Universitario de Matemática Multidisciplinar  
Universitat Politècnica de València,  
Cno. de Vera, 14, 46022 València, Spain.  
jlhueso@mat.upv.es

<sup>2</sup>Instituto de Matemática Pura y Aplicada  
Universitat Politècnica de València,  
Cno. de Vera, 14, 46022 València, Spain.  
eumarti@mat.upv.es

## An improvement of the accessibility of Steffensen's method

J. A. Ezquerro<sup>1</sup>, M. A. Hernández<sup>1</sup>, M. J. Rubio<sup>1</sup>, A. I. Velasco<sup>1</sup>

Many scientific and engineering problems can be brought in the form of a nonlinear equation  $F(x) = 0$ , where  $F$  is a nonlinear operator defined on a non-empty open convex subset  $\Omega$  of a Banach space  $X$  with values in  $X$ . In general, if the operator  $F$  is nonlinear, iterative methods are used to solve  $F(x) = 0$ . A very important aspect in the study of iterative methods is the choice of good initial approximations. In general, iterative methods usually converge once the initial approximations satisfy certain conditions (semilocal convergence).

In this work, for solving  $F(x) = 0$ , we consider one-point iterative methods with memory of the form ([2]):

$$\begin{cases} x_{-1}, x_0 \in \Omega, \\ x_{n+1} = \psi(x_{n-1}; x_n), \quad n \geq 0, \end{cases}$$

where  $\psi$  is an operator defined on a non-empty open convex subset  $\Omega$  of a Banach space  $X$  with values in  $X$ . In particular, we use iterative methods that do not use derivatives of the operator  $F$  in their algorithms.

In [1], from modified Newton's method and Steffensen's method, an hybrid iterative method is constructed for approximating the roots of nonlinear equations  $F(x) = 0$ , where the operator  $F$  is differentiable. In this work, we present a modification of the previous hybrid iterative method, which is based on the simplified secant method instead of modified Newton's method, that allows us to extend the solution of nonlinear equations to equations where the operator  $F$  is nondifferentiable. We analyse the semilocal convergence of the new hybrid iterative method and illustrate it with a numerical example.

**Keywords:** Simplified secant method, Steffensen's method, hybrid iterative method.

**MSC 2010:** 47H99, 65J15.

## References

- [1] J. A. EZQUERRO; M. A. HERNÁNDEZ; N. ROMERO; A. I. VELASCO, On Steffensen's method on Banach spaces. Enviado a *J. Comput. Appl. Math.*.
- [2] J. F. TRAUB, *Iterative methods for the solution of equations*. Prentice-Hall, Englewood Cliffs, New Jersey, 1964.

<sup>1</sup>Departamento de Matemáticas y Computación  
Universidad de La Rioja  
Calle Luis de Ulloa s/n, 26007 Logroño, España  
<jezquer><mahernan><mjesus.rubio>@unirioja.es,  
anabelmates@hotmail.com.ar

## On a family of two-step relaxed Newton-type methods

S. Amat<sup>1</sup>, S. Busquier<sup>2</sup>, Á. A. Magreñán<sup>3</sup>, N. Romero<sup>4</sup>

We have studied a variant of the classical two-step Newton-method combined with the relaxed Newton method for the approximation of nonlinear equations in Banach spaces. The method has the following form:

$$\begin{cases} x_0 \text{ given,} \\ y_n = x_n + \lambda_n \Gamma_n F(x_n), & 0 < \lambda_n \leq 1 \\ x_{n+1} = x_n - \lambda_n \Gamma_n F(y_n). \end{cases} \quad (1)$$

where  $\Gamma_n = [F'(x_n)]^{-1}$ . Notice that this method is free of any bilinear operator and in each iteration we only approximate an associated linear system. This method has been recently studied in [1] and [2]. In this talk we present a semilocal convergence result under  $\omega$ -conditioned divided differences for this method and some results about the order of convergence. Finally some numerical examples will show the improvement and advantages of the damped method compared with the usual one.

**Keywords:** Newton type methods, semilocal convergence, divided differences.

**MSC 2010:** 65B05, 47H17, 49M15.

## References

- [1] S. AMAT; C. BERMÚDEZ ; S. BUSQUIER ; S. PLAZA, On a third-order Newton-type method free of bilinear operators, *Numer. Linear Algebra Appl.*, **17**, 639–653 (2010).
- [2] S. AMAT; C. BERMÚDEZ ; S. BUSQUIER ; S. PLAZA, On two families of high order Newton type methods, *Applied Mathematics Letters*, **25** (12), 2209–2217 (2012).

<sup>1</sup>Departamento de Matemática Aplicada y Estadística  
Universidad Politécnica de Cartagena  
Paseo Alfonso XIII, nº52. 30203. Cartagena. España,  
sergio.amat@upct.es

<sup>2</sup>Departamento de Matemática Aplicada y Estadística  
Universidad Politécnica de Cartagena  
Paseo Alfonso XIII, nº52. 30203. Cartagena. España,  
sonia.busquier@upct.es

<sup>3</sup>Departamento de Matemáticas y Computación.  
Universidad de La Rioja.  
C/ Luis de Ulloa s/n. 26004 Logroño. España,  
alberto.magrenan@gmail.com

<sup>4</sup>Departamento de Matemáticas y Computación.  
Universidad de La Rioja.  
C/ Luis de Ulloa s/n. 26004 Logroño. España,  
natalia.romero@unirioja.es

## Una visión general sobre la convergencia del método de Newton amortiguado

José M. Gutiérrez<sup>1</sup>, Á. Alberto Magreñán<sup>1</sup>

En este trabajo se considera el método de Newton amortiguado

$$x_{n+1} = x_n - \lambda F'(x_n)^{-1} F(x_n), \quad \lambda \in (0, 1], \quad n \geq 0$$

para aproximar una solución de la ecuación  $F(x) = 0$ , siendo  $F$  un operador no lineal definido entre dos espacios de Banach. Se estudia la convergencia del método usando distintas técnicas tanto locales como semilocales: teoría de Kantorovich, relaciones de recurrencia,  $\alpha$  y  $\gamma$  teoría de Smale, etc. El objetivo es analizar la influencia del parámetro amortiguador  $\lambda$  en los resultados obtenidos.

**Keywords:** Damped Newton's method, Kantorovich theory,  $\alpha$ -theory

**MSC 2010:** 65J15, 47H10

## Referencias

- [1] I.K. ARGYROS; S. HILOUT, On the semilocal convergence of damped Newton's method. *Appl. Math. Comput.* <http://dx.doi.org/10.1016/j.amc.2012.09.011>
- [2] J. P. DEDIEU, *Points fixes, zéros et la méthode de Newton*. Mathématiques et Applications 54. Springer Verlag, Berlín, 2006.
- [3] L. V. KANTOROVICH; G. P. AKILOV, *Functional analysis*. Pergamon Press, Oxford, 1982.
- [4] P. D. PROINOV, New general convergence theory for iterative processes and its applications to Newton-Kantorovich type theorems. *J. Complexity* **26**, 3–42 (2010).
- [5] D. WANG; F. ZHAO, The Theory of Smale's Point Estimation and its Applications. *J. Comput. Appl. Math.* **60**, 253–269 (1995).

<sup>1</sup>Departamento de Matemáticas y Computación  
Universidad de La Rioja  
c/ Luis de Ulloa, s/n, 26004 Logroño  
{jmguti, angel-alberto.magrenan}@unirioja.es

## Aproximación de sistemas de ecuaciones no lineales y no estacionarios

Sergio Amat<sup>1</sup>, Sonia Busquier<sup>1</sup>, Pablo Pedregal<sup>2</sup>

En muchas aplicaciones nos encontramos con problemas no lineales. En el caso estacionario los métodos tipo Newton son los más usados. Estos métodos pueden verse dentro del método general de mínimos cuadrados. En esta charla presentaremos una generalización de este método para abordar la aproximación de problemas no estacionarios.

**Keywords:** Ecuaciones no lineales dependientes del tiempo, Mínimos cuadrados, Técnicas variacionales

**MSC 2010:** 65J15, 49M15

### Referencias

- [1] S. AMAT AND P. PEDREGAL, On an alternative approach for the analysis and numerical simulation of stiff ODEs, *Discrete and Continuous Dynamical Systems - Series A* **33**(4), 1275–1291 (2013).

<sup>1</sup>Departamento de Matemática Aplicada y Estadística  
Universidad Politécnica de Cartagena  
Campus Alfonso XIII (Spain).  
sergio.amat@upct.es, sonia.busquier@upct.es

<sup>2</sup>E.T.S. Ingenieros Industriales.  
Universidad de Castilla La Mancha.  
Campus de Ciudad Real (Spain).  
pablo.pedregal@uclm.es

## **Sobre unos métodos tipo secante de alto orden de convergencia y libres de derivadas**

**Candela, Vicente<sup>1</sup>; Trillo, Juan Carlos<sup>2</sup>**

En este trabajo presentamos una adaptación de los métodos que generalizan el método de la Secante y de Müller mediante el uso de polinomios de mayor grado. Es conocido que la generalización inmediata de estos dos métodos lleva a otros métodos no muy eficientes y con la restricción de tener su orden de convergencia limitado por 2. Nosotros introducimos una modificación que permite generar algoritmos cuyos ordenes forman una sucesión estrictamente creciente y no acotada a la vez que conservan el hecho de no utilizar derivadas.

**Keywords:** Secante, Müller, Razón Áurea

**MSC 2010:** 65H04, 65H05

<sup>1</sup>Departamento de Matemática Aplicada  
Universidad de Valencia  
C/ Doctor Moliner, s/n, 46100 Burjassot  
Vicente.Candela@uv.es

<sup>2</sup>Departamento de Matemática Aplicada y Estadística  
Universidad Politécnica de Cartagena  
Paseo Alfonso XIII, 52, 30203 Cartagena  
jctrillo@upct.es

## On the Application of Rates of Multiplicity for Finding Singular Roots of Nonlinear Equations.

Vicente F. Candela<sup>1</sup>, Rosa M. Peris<sup>1</sup>

Given  $f(x)$ , its *rate of multiplicity* of order  $p$  is defined as any function  $\mu(x)$  such that, close to any root or pole  $x^*$  of  $f(x)$ ,

$$\frac{|f(x)|}{|x - x^*|^{\mu(x^*)}}$$

is bounded, and  $\mu(x) = \mu(x^*) + O((x - x^*)^p)$ .

Usefulness of these rates consists of allowing to search for roots with no knowledge of its exact multiplicity. Thus, iterative methods such as Newton or higher order ones, get accelerated when using these rates. Most methods needing the exact multiplicity can be adapted by means of rates in order to get efficient and robust versions with a moderate increase of computational cost.

In this talk, we will analyze the properties of the rates of multiplicity, their relationship with other fundamental concepts in iterative methods, such as logarithmic convexity or elasticity, and techniques to construct higher order rates while avoiding the evaluation of high order derivatives.

Some iterative methods will be considered through the scope of rates, and theoretical results will help to study convergence of methods for multiple roots by means of their associated rates.

Finally, some examples will illustrate the performance of the proposed methods even in case of noninteger multiplicity (appearing in fractional analysis) or poles (usual in complex problems).

**Keywords:** iterative Newton-type methods, nonlinear equations, ill conditioning, multiple roots, poles, local convergence, stability.

**MSC 2010:** 65H05, 65H10, 65B99

## References

- [1] V. F. CANDELA, A. MARQUINA, Recurrence relations for rational cubic methods I: The Halley method, *Computing*, **44**, 169-184, (1990).
- [2] V. F. CANDELA, A. MARQUINA, Recurrence relations for rational cubic methods II: The Chebyshev method, *Computing*, **45**, 355-367, (1990).

- [3] C. CHUN, B. NETA, A third order modification of Newton's method for multiple roots, *Appl. Math. Comput.*, **211**, 474-479, (2011).
- [4] S. KUMAR, V. KANWAR, S. SINGH, On some modified families of multipoint iterative methods for multiple roots of nonlinear equations, *Appl. Math. Comput.*, **218**, 7382-7394, (2012).
- [5] J.M. ORTEGA, W.C. RHEINBOLD, *Iterative Solution of Nonlinear Equations in Several Variables*. Academic Press, Reading, MA, 1970.
- [6] R. PERIS, A. MARQUINA, V. CANDELA, The convergence of the perturbed Newton method and its application for ill-conditioned problems, *Appl. Math. Comput.*, **218**, 2988-3001, (2011).
- [7] R. PERIS, V. CANDELA, F. POTRA, Iterative methods for ill-conditioned problems: the function rate of multiplicity. En *SIAM Conference on Applied Linear Algebra*, Valencia, 2012.
- [8] L.B. RALL, Convergence of Newton process to multiple solutions, *Numer. Math.*, **9**, 23-37 (1966)
- [9] S. WEERAKOON, TGI FERNANDO, A variant of Newton's method with accelerated third-order convergence, *Applied Math Letters*, **13** (8), 87-93 (2000).

<sup>1</sup>Departamento Matemática Aplicada  
 Universidad de Valencia  
 Dr. Moliner 50, 46100 Burjassot  
 peris, candela@uv.es

## Strategies for solutions of ill conditioned nonlinear equations

Rosa M. Peris<sup>1</sup>, Vicente F. Candela<sup>1</sup>

Computational efficiency of iterative methods to solve nonlinear equations of the type  $F(x) = 0$  gets handicapped in case of ill conditioned problems. A well known result states that high order methods, such as Newton, decay to first order in the presence of multiple roots. However, even simple roots may be problematic, if the derivative  $F'(x^*)$  (being  $x^*$ , the root  $F(x^*) = 0$ ) is small enough, or if  $x^*$  is too close to other different roots.

Most methods are analyzed in ideal conditions, where the equation is regular enough as not to introduce any additional difficulty. From a local point of view (that is, from a neighbourhood of the simple root), there is always a region where the method can be considered in these ideal conditions. The main problem arises when that regular region can be hardly reached, because it is difficult to distinguish different roots or the function is extremely flat around the root. This problem gets worse when solving nonlinear systems, due to the fact that roots interact and are difficult to separate.

In this talk, we deal with these ill conditioned cases. We propose strategies in order to reduce the harmful effects above mentioned (strategies which, in the other side, may be used even in the most positive conditions), based on the regularization of the iteration. Criteria for these choices are also provided, following the properties of the equation, in such a way that the methods are adapted to the problem instead of the usual techniques which try to adapt the problem to the methods.

The theoretical analysis will be illustrated through examples.

**Keywords:** iterative methods, nonlinear equations, Newton method, ill conditioning, local convergence, stability.

**MSC 2010:** 65H05, 65H10, 65B99

## References

- [1] V. F. CANDELA, A. MARQUINA, Recurrence relations for rational cubic methods I: The Halley method, *Computing*, **44**, 169-184, (1990).
- [2] V. F. CANDELA, A. MARQUINA, Recurrence relations for rational cubic methods II: The Chebyshev method, *Computing*, **45**, 355-367, (1990).

- [3] V. KANTOROVICH, G.P. AKILOV, *Functional Analysis*. Pergamon Press, Oxford, 1982.
- [4] J.M. ORTEGA, W.C. RHEINBOLD, *Iterative Solution of Nonlinear Equations in Several Variables*. Academic Press, Reading, MA, 1970.
- [5] R. PERIS, A. MARQUINA, V. CANDELA, The convergence of the perturbed Newton method and its application for ill-conditioned problems, *Appl. Math. Comput.*, **218**, 2988-3001, (2011).
- [6] R. PERIS, V. CANDELA, F. POTRA, Iterative methods for ill-conditioned problems: the function rate of multiplicity. En *SIAM Conference on Applied Linear Algebra*, Valencia, 2012.
- [7] W.C. RHEINBOLD, *Methods for Solving Systems of Nonlinear Equations*. SIAM, Philadelphia, 1966.
- [8] S. WEERAKOON, TGI FERNANDO, A variant of Newton's method with accelerated third-order convergence, *Applied Math Letters*, **13** (8), 87-93 (2000).

<sup>1</sup>Departamento Matemática Aplicada  
 Universidad de Valencia  
 Dr. Moliner 50, 46100 Burjassot  
 peris, candela@uv.es

## Dinámica de la familia de King de métodos iterativos\*

Alicia Cordero<sup>1</sup>, Javier García<sup>2</sup>, Juan R. Torregrosa<sup>1</sup>, María P. Vassileva<sup>2</sup>

En este trabajo se analiza la dinámica de la familia de King (véase [2]) de métodos iterativos para la aproximación de raíces de ecuaciones no lineales  $f(x) = 0$ , cuya expresión iterativa es

$$x_{k+1} = y_k - \frac{f(x_k) + (2 + \beta)f(y_k)}{f(x_k) + \beta f(y_k)} \frac{f(y_k)}{f'(x_k)}, \quad (1)$$

donde  $y_k$  es el iterado del método de Newton.

Desde el punto de vista numérico, el comportamiento dinámico de la función racional asociada a un método iterativo proporciona información interesante acerca de su estabilidad y fiabilidad. En estos términos, Varona en [4] describe la dinámica asociada a diferentes métodos iterativos conocidos. Más recientemente, en [3] Neta et al. estudian el comportamiento dinámico, sobre polinomios de distintos grados, de métodos iterativos para raíces múltiples.

Partiendo del operador racional asociado a la familia (1) que actúa sobre polinomios cuadráticos genéricos, analizamos los diferentes planos de parámetros encontrados proporcionando una visión general de los distintos comportamientos de los miembros que componen la familia. Podemos encontrar un análisis previo de esta familia en el trabajo de Amat et al. [1].

La riqueza dinámica de esta familia es particularmente interesante encontrando comportamientos análogos a los observados en el conjunto de Mandelbrot.

**Keywords:** Ecuaciones no lineales, Métodos iterativos, Análisis dinámico

**MSC 2010:** 65H05, 37F10, 37H20

## Referencias

- [1] S. AMAT; S. BUSQUIER; S. PLAZA, Dynamics of the King and Jarratt iterations. *Aequationes Mathematicae* **69**, 212–223 (2005).
- [2] R.F. KING, A family of fourth-order methods for nonlinear equations. *SIAM Journal of Numerical Analysis* **10** (5), 876–879 (1973).

---

\*This research was supported by Ministerio de Ciencia y Tecnología MTM2011-28636-C02-02 and FONDOCYT República Dominicana.

- [3] B. NETA; M. SCOTT; C. CHUN, Basin attractors for various methods for multiple roots. *Applied Mathematics and Computation* **218**, 5043–5066 (2012).
- [4] J.L. VARONA, Graphic and numerical comparison between iterative methods. *Mathematical Inteligencer* **24** (1), 37–46 (2002).

<sup>1</sup>Instituto de Matemáticas Multidisciplinar,  
Universidad Politécnica de Valencia,  
Camino de Vera, s/n, 46022 Valencia  
acordero@mat.upv.es, jr torre@mat.upv.es

<sup>2</sup>Area de Ciencia Básica,  
Instituto Tecnológico de Santo Domingo (INTEC),  
Avd. Los Próceres, Galá, Santo Domingo, República Dominicana  
javiermaimo@hotmail.com, marip@intec.edu.do

## **Evaluación numérica de ceros complejos de funciones especiales**

**Javier Segura**<sup>1</sup>

Describimos métodos para la evaluación eficiente de ceros complejos de soluciones de ecuaciones diferenciales ordinarias de segundo orden lineales y su aplicación para la evaluación numérica de los ceros de funciones especiales. Los ceros de las EDOs lineales y homogéneas de segundo orden siguen trayectorias en el plano complejo dadas por las líneas de anti-Stokes. Consideramos la aproximación de Liouville-Green para ODEs del tipo  $w''(z) + A(z)w(z) = 0$  siendo  $A(z)$  una función meromorfa, y describimos las propiedades cualitativas de las líneas de anti-Stokes para esta aproximación. Basándonos en esta descripción cualitativa, construimos un método de orden cuatro capaz de evaluar eficientemente los ceros complejos siguiendo las líneas de anti-Stokes aproximadas. El funcionamiento del método se ilustra con algoritmos para la evaluación de los ceros complejos de algunas funciones especiales (funciones de Bessel, funciones del cilindro parabólico y polinomios de Bessel).

**Keywords:** Funciones especiales, EDOs lineales, ceros complejos

**MSC 2010:** 33F05, 65H05, 30E09

<sup>1</sup>Departamento de Matemáticas, Estadística y Computación  
Universidad de Cantabria  
Avda. Los Castros, 39005-Santander, España  
`javier.segura@unican.es`

## Aproximación de sistemas Hamiltonianos usando una nueva técnica variacional

Sergio Amat<sup>1</sup>, M<sup>a</sup> J. Légaz<sup>1</sup>, Pablo Pedregal<sup>2</sup>

Se comenzará con una introducción a los problemas Hamiltonianos y su aproximación por medio de métodos simplécticos. Seguidamente presentaremos una nueva perspectiva variacional para aproximar este tipo de problemas. Se minimizará un funcional de error asociado de forma natural a la ecuación y donde se penalizará la no conservación de la energía. Daremos de forma exacta una dirección de descenso y se darán resultados de convergencia. Finalmente, se presentarán experimentos numéricos donde testaremos esta nueva alternativa.

**Keywords:** Problemas Hamiltonianos, Técnicas variacionales, Métodos simplécticos, Conservación de la Energía

**MSC 2010:** 65J15, 49M15

### Referencias

- [1] S. AMAT AND P. PEDREGAL P., On an alternative approach for the analysis and numerical simulation of stiff ODEs, *Discrete and Continuous Dynamical Systems - Series A* **33**(4), 1275–1291 (2013).

<sup>1</sup>Departamento de Matemática Aplicada y Estadística  
Universidad Politécnica de Cartagena  
Campus Alfonso XIII (Spain).  
sergio.amat@upct.es, sonia.busquier@upct.es

<sup>2</sup>E.T.S. Ingenieros Industriales.  
Universidad de Castilla La Mancha.  
Campus de Ciudad Real (Spain).  
pablo.pedregal@uclm.es

## Tracking multiplicities

Marc Giusti<sup>1</sup>, Jean-Claude Yakoubsohn<sup>2</sup>

It is well known that the computation of the multiplicity and the approximation of isolated multiple roots of polynomial systems is a difficult problem. In recent years, there has been an increase of activity in this area. One goal is to translate the theoretical background developed in the last century on the theory of singularities in terms of computation and complexity. This talk presents several different views that seem relevant to address the following issues: decide of the multiplicity of a root and/or determine the number of roots in a ball, approximate fastly a multiple root and give complexity results for such problems. Finally, we propose a new method to determine a regular system admitting the same root as the initial singular one.

<sup>1</sup>Laboratoire LIX  
Campus de l'École Polytechnique  
Ecole Polytechnique  
F-91128 PALAISEAU CEDEX  
`Marc.Giusti@polytechnique.fr`

<sup>2</sup>Department IMT MIP  
Université Paul Sabatier  
118 Route de Narbonne  
Toulouse 3, France  
`yak@mip.ups-tlse.fr`