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Congreso RSME 2013

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New examples of Hopf algebras with nonzero integral

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Mier 23, 18:00 - 18:25, Aula 8 – Mercedes Siles:
Centros de álgebras asociadas a grafos

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Mier 23, 19:00 - 19:25, Aula 8 – Pascual Jara:
Koszul pairs

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Superdecomposable pure-injective modules over tubular algebras

Mike Prest[1]

Let A be a tubular finite-dimensional algebra (over a countable field). It has been known for a long time that such an algebra has Krull-Gabriel dimension ∞, hence has continuum many indecomposable pure-injective modules (and the finite-dimensional indecomposables account for only countably many of these) but it was an open question whether there are superdecomposable pure-injectives (that is, pure-injective modules without any indecomposable summands). This was answered, positively, by Richard Harland in his PhD thesis [1]. Indeed, he showed that for every positive irrational real number r there is a superdecomposable module of slope r. I will discuss this and give some of the ideas that go into the proof.

Keywords: tubular algebra, superdecomposable pure-injective, pp formula

MSC 2010: 16G20, 03C60

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Frobenius and separable wreath extensions

Daniel Bulacu[1], **Blas Torrecillas**[2]

We study Frobenius and separable for algebras in monoidal categories. A wreath is a monad in the Eilenber-Moore category associate to a 2-category. We obtain the characterization of Frobenius and separable for wreath extensions.

Keywords: Frobenius extension, separable extensions, 2-categories, wreath

MSC 2010: 16W30

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Resolving subcategories of modules of finite projective dimension

Lidia Angeleri[1], **Manuel Saorín**[1]

Let R be a Noetherian commutative ring and denote by $Mod - R$ and $mod - R$ its categories of arbitrary and finitely generated modules, respectively. Recent results by Alonso-Jeremías-Saorín [1] and Angeleri-Pospisil-Stovicek-Trlifaj [3] give classifications of the compactly generated t-structures of the derived category $D(R)$ and the (co)tilting classes in the category $Mod - R$. In this talk we will show that, as an application of the first mentioned result, one obtains a classification of the resolving subcategories of $mod-R$ consisting of modules of finite projective dimension. When n is a fixed natural number and one restricts to consider only resolving subcategories of $mod - R$ consisting of modules of projective dimension less or equal than n, then one recovers one of the main results in [3].

Recall that if $\mathcal{A}$ is an abelian category with enough projectives, then a full subcategory $\mathcal{R} \subseteq \mathcal{A}$ is resolving when it contains the projective objects and is closed under taking direct summands, extensions and kernels of epimorphisms. The main result of this talk is the following, where $E_i(R)$ denotes the i-th term of the minimal injective resolution of R:

THEOREM.- There are bijective correspondences between

(i) resolving subcategories of $mod - R$ with modules of finite projective dimension

(ii) filtrations by supports Φ with $\Phi(i) = SpecR$ for $i < 0$ and $AssE_i(R) \cap \Phi(i) = \emptyset$ for $i \geq 0$;

(iii) compactly generated t-structures $(\mathcal{U}, \mathcal{V})$ in $\mathcal{D}(R)$ with $R[1]$ in their heart.

We will then connect the theorem with recent results in the same direction by Dao and Takahashi [2] and, if time permits, we will show how the result gives a new perspective on the finitistic dimension of R. In particular, it gives new necessary and sufficient conditions for the finiteness of this dimension.

Keywords: Resolving subcategory, t-structure, filtration by supports

MSC 2010: 13Dxx, 16Exx

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A note on the construction of finitely injective modules

Pedro A. Guil Asensio[1], **Manuel Cortés-Izurdiaga**[2], **Blas Torrecillas**[2]

Let R be a ring with unit. A left R-module M is called finitely injective if each finite subset of M is contained in an injective submodule. In the talk, we shall develop a technique to construct finitely injective modules which are non trivial, in the sense that they are not direct sums of injective modules. As a consequence, we shall prove that a ring R is left noetherian if and only if each finitely injective left R-module is trivial, thus answering an open question posed by Salce.

Keywords: Noetherian rings; finitely injective modules

MSC 2010: 16D50, 16D70

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Regularidad en el Semigrupo de Cuntz

Ramon Antoine[1], Francesc Perera[1], Hannes Thiel[2]

El estudio y clasificación de C*-álgebras nucleares, conocido como el programa de clasificación de Elliott, se aplica con éxito dentro de la subcategoría de álgebras $\mathcal{Z}$-estables dónde $\mathcal{Z}$ es el álgebra de Jiang-Su (véase [2] y [3]). Ésta $\mathcal{Z}$-estabilidad aporta una determinada regularidad al tipo de álgebras que se estudian, evitando así ciertos contraejemplos a la conjetura de Elliott sobre dicha clasificación. A. Toms y W. Winter [4] estudian las C*- álgebras $\mathcal{D}$ llamadas "strongly self absorbing" que generalizan y tienen propiedades similares a $\mathcal{Z}$ y que a su tiempo dotan también de cierta regularidad las correspondientes familias de álgebras $\mathcal{D}$-estables.

En la última década, un invariante algebraico, llamada el Semigrupo de Cuntz, ha pasado al frente de dicho programa de clasificación como una herramienta indispensable. Denotado por $\mathrm{Cu}(A)$, pertenece a una categoría de semigrupos (Cu) con una estructura ordenada muy rica. En éste trabajo, y basándonos en ciertas ideas de L. Robert en [1], traducimos la situación anteriormente descrita a la categoría Cu. Aquí, el papel de las álgebras fuertemente auto-absorbentes corresponderá a ciertos semi-anillos, que dotaran a los semigrupos de Cuntz de álgebras $\mathcal{D}$-estables de una particular estructura de semi-módulo. En éste proceso, aventuramos una posible definición de producto tensorial en la categoría Cu y observamos cómo resultados analíticos disponen de su correspondiente y natural equivalente en Cu.

Keywords: Semigrupo de Cuntz, C*-álgebra, Teoría K

MSC 2010: 46L06, 46L35, 46L80, 20M10

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Approximations and locally free modules

Jan Trlifaj[1]

In [3], Eklof and Shelah have established a limit to the approximation theory of modules. They have shown that the statement "Whitehead groups form a precovering class" is independent of ZFC. More recently, in [1], [6], and [7], flat Mittag-Leffler modules over countable non-perfect rings were shown not to form a precovering class in ZFC. The latter fact has remarkable consequences for the cohomology of Drinfeld vector bundles, cf. [2] and [4].

By finding a new proof of the main result from [6], we discover a connection of the non-precovering phenomenon with infinite dimensional tilting theory (in the sense of [5, Part III], for example). In particular, we trace the phenomenon to all countable hereditary artin algebras A of infinite representation type: there, the class of all locally Baer A-modules is not precovering [8].

Keywords: module approximations, locally free modules, tilting theory.

MSC 2010: 16D70; 03E75, 13F05, 16G10, 18G25.

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Gorenstein homological algebra in the category of discrete modules over profinite groups*

E. Enochs[1], **J. R. García Rozas**[2] **L. Oyonarte**[2] **B. Torrecillas**[2]

Gorenstein homological algebra was introduced in categories of modules. But it has proved to be a fruitful way to study various other categories such as categories of complexes and of sheaves.

In this talk, we initiate the research of relative homological algebra in categories of discrete modules over profinite groups. This seems appropriate since (in some sense) the subject of Gorenstein homological algebra had its beginning with Tate homology and cohomology over finite groups. We prove that if the profinite group has virtually finite cohomological dimension then every discrete module has a Gorenstein injective envelope, a Gorenstein injective cover, a torsion free cover and study various cohomological dimensions relative to Gorenstein injective discrete modules.

Keywords: Gorenstein category, profinite group, group cohomology

MSC 2010: 16E65, 20E18

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Locally torsion-free quasi–coherent sheaves*

Sinem Odabaşı[1]

Let X be an arbitrary scheme. The category $\mathfrak{Qcoh}(X)$ of quasi–coherent sheaves on X is known that admits arbitrary direct products. However their structure seems to be rather mysterious. In this talk we will describe the structure of the product object of a family of locally torsion-free objects in $\mathfrak{Qcoh}(X)$, for X an integral scheme. Several applications are provided. For instance it is shown that the class of flat quasi–coherent sheaves on a Dedekind scheme X is closed under arbitrary direct products, and that the class of all locally torsion-free quasi–coherent sheaves induces a hereditary torsion theory on $\mathfrak{Qcoh}(X)$. Finally torsion-free covers are shown to exist in $\mathfrak{Qcoh}(X)$.

Keywords: torsion-free quasi–coherent sheaf, integral scheme, torsion theory, cover

MSC 2010: 13D30,18E40,18F20 (primary),14F05,18A30 (secondary)

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Endomorphisms of $\aleph_n$-free modules over the p-adic integers

Lutz H. Strüngmann[1]

It has been a long tradition in module theory and in particular in Abelian group theory to construct large modules/groups that have a prescribed endomorphism ring. In ordinary set-theory ZFC these constructions mostly used a combinatorial principle due to Saharon Shelah and called *Black-Box*. The resulting modules or groups had additional freeness properties, e.g. in the torsion-free case every countable subset was contained in a free submodule. Whenever a stronger freeness-condition was required replacing countable by larger cardinalities one had to assume additional set-theoretic axioms to carry out the construction. Examples are the diamond principle in Gödel's universe or Martin's axiom.
In this talk we will present a recent result showing the existence of large $\aleph_n$-free J_p-modules with prescribed dual/endomorphism ring in ZFC. Necesaarily these modules have to be verylarge to ensure the freeness-property in ZFC. A new combinatorial principle called $\aleph_n$*-free Black Box* is used.
This is joint work with Rüdiger Göbel and Saharon Shelah from [1].

Keywords: abelian groups, endomorphism rings, p-adic integers

MSC 2010: 20K15

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A question about module extensions in the context of homotopical localizations of spaces

José L. Rodríguez[1]

Let R be any commutative ring with 1, and consider extensions $\beta : 0 \to B \to E \to G \to 0$ of R-modules such that the canonical morphism $\beta^* : \mathrm{Hom}(B, B) \to \mathrm{Ext}(G, B)$ is an isomorphism. This situation emerges in the study of localizations of Eilenberg–Mac Lane spaces $K(G, n)$, which will be treated in detail during the talk.

If R is the ring $\mathbf{Z}$ of integers and G is a Prüfer group $\mathbf{Z}(p^\infty)$, then the localizations of $K(G, n)$ take the form $K(B, n+1)$, where B is a reduced abelian group satisfying $\mathrm{Hom}(B, B) \cong \mathrm{Ext}(\mathbf{Z}(p^\infty), B)$. If furthermore the group B is Ext-p-complete, then it is either isomorphic to $\mathbf{Z}/p^k$ or to the ring of p-adic integers. What is left open is to determine whether there are other examples, if B is non Ext-p-complete.

This is a joint work with Carles Casacuberta which will be included in [1].

Keywords: Homotopical localization, Ext-p-complete groups, E-rings, module extensions.

MSC 2010: Primary: 55P60; 55P20; Secondary: 20K45; 20K35, 13B35.

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Semilocal categories, local functors and applications

Alberto Facchini[1]

A ring R is *semilocal* if $R/J(R)$ is semisimple artinian, that is, a finite direct product of rings of matrices over division rings. A preadditive category $\mathcal{A}$ is a *null* category if all its objects are zero objects. A preadditive category is *semilocal* if it is non-null and the endomorphism ring of every non-zero object is a semilocal ring. The following are examples of full semilocal subcategories of the category Mod-R of all right modules over an associative ring R:

1. the full subcategory of all artinian right R-modules (Camps and Dicks);
2. the full subcategory of all finitely generated R-modules, for R a semilocal commutative ring (Warfield);
3. the full subcategory of all finitely presented modules right R-modules, for R a semilocal ring (Facchini and Herbera);
4. the full subcategory of all serial modules of finite Goldie dimension (Herbera and Shamsuddin);
5. the full subcategory of all modules of finite Goldie dimension and finite dual Goldie dimension (Herbera and Shamsuddin).

An additive functor $F\colon \mathcal{A} \to \mathcal{B}$ between preadditive categories $\mathcal{A}$ and $\mathcal{B}$ is said to be a *local functor* if, for every morphism $f\colon A \to A'$ in $\mathcal{A}$, $F(f)$ isomorphism in $\mathcal{B}$ implies f isomorphism in $\mathcal{A}$. This notion must not be confused with the notion of *isomorphism reflecting* functor: for every A, A' objects of $\mathcal{A}$, $F(A) \cong F(A')$ implies $A \cong A'$.

We will present the interplay between the concepts of semilocal category, local functor, Jacobson radical of the category and maximal ideals. Our may concern will be the study of the canonical functor $\mathcal{A} \to \mathcal{A}/\mathcal{I}_1 \times \cdots \times \mathcal{A}/\mathcal{I}_n$, where $\mathcal{A}$ is a preadditive category and $\mathcal{I}_1, \ldots, \mathcal{I}_n$ are ideals of $\mathcal{A}$. We will consider and characterize the case where this canonical functor is local. An application will be given.

Keywords: Semilocal ring, Semilocal category, Local functor

MSC 2010: 16D70, 16D90, 18E05

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Inversion height and crossed products

Dolors Herbera[1], **Javier Sánchez**[2]

Let X be a set with at least two elements, and let k be any commutative field. We prove that the inversion height of the embedding $k\langle X\rangle \hookrightarrow D$, where D denotes the universal division ring of fractions of the free algebra $k\langle X\rangle$, is infinite. Therefore, if H denotes the free group on X, the inversion height of the embedding of the group algebra $k[H]$ into the Mal'cev Neumann series ring is also infinite. This answers in the affirmative a question posed by Neumann in 1949 [3], and reformulated again in [1].

The case of an infinite set was already settled by Reutenauer [4]. Recall that, for a finite set X, there are many embeddings of a free algebra over an infinite set, call it S, inside $k\langle X\rangle$. Essentially, we show that if $k\langle X\rangle$ can be given a *Lie Algebra crossed product* structure over such an S then the universal division rings of fractions of S can be embedded inside the universal division rings of fractions of $k\langle X\rangle$ and the embedding preserves the inversion height. We settle Neumann's question showing that such crossed product structures are relatively frequent.

We also show that the same type of argument can be done seeing D as the universal field of fractions of the group algebra over the free group. In this case, a suitable *group crossed product* structure allows us to prove the same kind of result.

Having infinite inversion height is not a distinctive feature of the free field. We give an infinite family of examples of non-isomorphic division ring of fractions of $k\langle X\rangle$ with infinite inversion height.

The results we present in this talk are part of the preprint [2].

Keywords: Division ring, inversion height, crossed product, free field

MSC 2010: 16K40, 16S35

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New examples of Hopf algebras with nonzero integral

Juan Cuadra[1]

The Haar measure on a compact group G induces a linear functional $\int$ on the Hopf algebra of representative functions on G. Hochschild observed that the invariance property of the Haar measure reads as a condition on $\int$ that can be expressed in Hopf algebraic terms. Later, Sweedler defined the notion of integral for arbitrary Hopf algebras by mean of this condition. Hopf algebras having a nonzero integral are called co-Frobenius.

The quantized coordinate algebra $\mathcal{O}_q(G)$ of a simple algebraic group G is one of the most important examples of co-Frobenius Hopf algebras. When q is a root of unity, the coordinate algebra $\mathcal{O}(G)$ of G is a central Hopf subalgebra of $\mathcal{O}_q(G)$ and $\mathcal{O}_q(G)$ is finitely generated (and free) as a module over $\mathcal{O}(G)$. Moreover, $\mathcal{O}(G)$ coincides with the Hopf socle of $\mathcal{O}_q(G)$. Based on this example, Andruskiewitsch and Dăscălescu asked in [*Co-Frobenius Hopf algebras and the coradical filtration*. Math. Z. **243** (2003), 145-154] whether any co-Frobenius Hopf algebra is finitely generated as a module over its Hopf socle.

In this talk we will introduce a family of co-Frobenius Hopf algebras that are not finitely generated over their Hopf socles, answering so in the negative this question. Concretely, we will construct a Hopf algebra H over a field depending on a primitive n-th root of unity, a non-empty set I, and a family $(q_i)_{i\in I}$ of nonzero scalars. We will prove that H is finitely generated over its Hopf socle if and only if there is a finite subset J of I such that $q_i^n = 1$ for all $i \in I\backslash J$ and q_j is an m_j-th root of unity for all $j \in J$.

The results that will be presented are part of a joint work with N. Andruskiewitsch (National University of Córdoba, Argentina) and P. Etingof (Massachusetts Institute of Technology, USA). Arxiv:1206.5934.

Keywords: Hopf algebra, quantum group, integral

MSC 2010: 16T05, 17B37

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Cohomology of algebras over weak Hopf algebras*

Ramón González Rodríguez[1]

The aim of this talk is to present the Sweedler cohomology for a cocommutative weak Hopf algebra H living in a strict symmetric monoidal category where every idempotent morphism splits. We show that the second cohomology group classifies completely weak crossed products, with a common preunit, of H over a commutative left H-module algebra A. The results that will be presented are part of a joint work with J.N. Alonso and J.M. Fernández (see [1] and [2]).

Keywords: Weak Hopf algebra, Sweedler cohomology, weak crossed products

MSC 2010: 18D10, 16W30

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Centralizer constructions and Yetter-Drinfeld modules in a weak context

J.N. Alonso Álvarez[1], **R. González Rodríguez**[2], **C. Soneira Calvo**[3]

The notion of weak braided Hopf algebra and that of weak Yang-Baxter operator were introduced by the authors in [1] to extend Radford's theory to projections of weak Hopf algebras in a strict symmetric monoidal category where every idempotent morphism splits. Specifically, there exists a categorical equivalence between the category of projections associated to a weak Hopf algebra H and the category ${}_H^H\mathcal{YD}$ of left-left Yetter-Drinfeld modules over H.

The center construction, introduced independently by Drinfeld (unpublished), Joyal and Street [6] and Majid [8], associates to a monoidal category $\mathcal{C}$ a braided monoidal category $\mathcal{Z}(\mathcal{C}))$. It is known that for a for a Hopf algebra H the center of the category of left H-modules is equivalent to ${}_H^H\mathcal{YD}$ [7].

In this talk we introduce various centralizer constructions and establish a monoidal equi- valence with the category ${}_D^D\mathcal{YD}$ of Yetter-Drifeld modules over a weak braided Hopf algebra D. Subsequently we apply the result to the calculus of centers in module categories.

Keywords: weak (braided) Hopf algebra, Yetter-Drinfeld module, center

MSC 2010: 18D10, 16T05, 16T25, 81R50

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An alternative perspective on projectivity of modules.

Christopher Holston[1], **Sergio R. López-Permouth**[1], **Joseph Mastromatteo**[1], **José E. Simental-Rodriguez**[2]

We approach the analysis of the extent of the projectivity of modules from a fresh perspective as we introduce the notion of relative subprojectivity. A module M and is said to be N-subprojective if for every epimorphism $g : B \to N$ and homomorphism $f : M \to N$, there exists a homomorphism $h : M \to B$ such that $gh = f$. For a module M, the subprojectivity domain of M is defined to be the collection of all modules N such that M is N-subprojective. We consider, for every ring R, the subprojective profile of R, namely, the class of all subprojectivity domains for R modules. We show that the subprojective profile of R is a semilattice, and consider when this structure has coatoms or a smallest element. Modules whose subprojectivity domain is smallest possible will be called subprojectively poor (sp-poor) or projectively indigent (p-indigent) and those with co-atomic subprojectivy domain are said to be maximally subprojective. This work is a natural continuation to recent papers that have embraced the systematic study of the injective, projective and subinjective profiles of rings.

Keywords: projective modules, injective modules, ring profiles

MSC 2010: 16D40, 16D50

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Morita base change in Hopf-(co)cyclic (co)homology

Laiachi El Kaoutit [1], Niels Kowalzig[2]

Given two Morita equivalent Lie algebroids, it is well known from literature that their cohomologies coincide up to isomorphisms. The same result holds true for Poisson manifolds and their Poisson homologies. On the other hand, as in the case of Lie algebra, Lie algebroid also has an universal enveloping ring which instead admits a structure of cocommutative (left) Hopf algebroid. It turns out that the (co)homology of any Lie algebroid is identified with the cyclic homology of its universal enveloping ring. Thus, a natural question arise concerning Morita invariance of (co)cyclic (co)homologies between two cocommutative (left) Hopf algebroids. Obviously, an answer to this question will automatically leads to Morita invariance at the level of Lie algebroids and perhaps a more conceptual and algebraic proof of the above geometric results. Unfortunately, up to now, a complete and satisfactory answer to that question is far from being clear.

In this talk we give a naive answer by considering Morita invariance of cyclic homologies between two (left) Hopf algebroids where one is obtained form the other using the Morita base change. In our main application we consider the Morita equivalence between the algebra of complex-valued smooth functions on the classical 2-torus and the coordinate algebra of the noncommutative 2-torus with rational parameter. We then construct a Morita base change left Hopf algebroid over this noncommutative 2-torus and show that its cyclic homology can be computed by means of the homology of the Lie algebroid of vector fields on the classical 2-torus.

Keywords: Morita equivalence; cyclic homology; Hopf algebroids; vector bundles; Lie algebroids; noncommutative tori.

MSC 2010: Primary 16D90, 16E40, 16T05; Secondary 18D10, 16T15, 19D55, 58B34

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Centros de álgebras asociadas a grafos

Mercedes Siles Molina[1]

La noción de centro de un álgebra asociativa juega un papel importante en Álgebra. Aparece directamente relacionada con los grupos de cohomología de dimensión cero y también con los de dimensión 1, es decir, las derivaciones. Además, estas están estrechamente relacionadas con el centro, ya que, cualquiera que sea el álgebra asociativa A, el álgebra de Lie $A^-/Z(A)$ es isomorfa al álgebra de Lie de las derivaciones internas de A (aquí, A^- denota la antisimetrizada de A). Esto significa que el estudio del centro puede considerarse como el primer paso en el estudio de las derivaciones de un álgebra.

En este trabajo, realizado junto con María Guadalupe Corrales García, Dolores Martín Barquero, Cándido Martín González y José Félix Solanilla Hernández, hemos estudiado el centro de las álgebras de caminos, el de las álgebras de caminos de Cohn, y hemos obtenido cotas inferiores y superiores para el centro de un álgebra de caminos de Leavitt. Además, queda perfectamente determinado el centro de un álgebra de caminos de Leavitt prima.

Hay, además de las que hemos dado en el primer párrafo, otras razones que nos han llevado a abordar el problema del centro en álgebras de caminos: relacionar la simplicidad de tales álgebras, que son asociativas, con la de las álgebras de Lie asociadas, así como tratar de caracterizar los elementos del centro en términos de propiedades de los elementos que constituyen el grafo subyacente. Es esta una peculiaridad que hace "agradable" el trabajar con álgebras asociadas a grafos: el que propiedades algebraicas se reflejan en el grafo y, recíprocamente, propiedades del grafo encuentran su equivalente en el álgebra.

Keywords: Álgebra de caminos, Álgebra de caminos de Cohn, Álgebras de caminos de Leavitt, centro.

MSC 2010: 16D70

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Sobre álgebras de color de Lie split

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Las álgebras de color de Lie son una generalización de las superálgebras de Lie, y por tanto de las álgebras de Lie, y tienen un importante papel en Física de particulas. Con la intención de estudiar la estructura de un álgebra de color de Lie arbitraria (sin restricciones sobre su dimensión o sobre su cuerpo base), introduciremos las álgebras de color de Lie split como la extensión natural de las álgebras de Lie split y las superálgebras de Lie split.

Desarrollando técnicas de conexiones de raíces en este tipo de estructuras, mostramos que cualquiera de estas álgebras L es de la forma $L = \mathcal{U} + \sum_j I_j$ siendo $\mathcal{U}$ un subespacio de la subálgebra (graduada) abeliana H y cualquier I_j un ideal (graduado) bien definido de L tal que $[I_j, I_k] = 0$ si $j \neq k$.

Bajo ciertas condiciones podremos caracterizar la simplicidad de L y mostraremos que L es la suma directa de la familia de sus ideales (graduados) minimales, siendo cada uno un álgebra de color de Lie split simple.

Keywords: álgebra de color de Lie, raíz, teoría de estructura

MSC 2010: 17B75, 17B65, 17B05, 17B20

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Koszul pairs*

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Koszul rings were introduced by Beilinson, Ginzburg and Sorgel as a natural generalization of Koszul algebras. In order to investigate some homological properties of this class of graded rings we introduce Koszul pairs: consisting on an algebra A and a coalgebra C satisfying certain compatibility properties.

The main examples of Koszul pairs are associated to certain braided bialgebras in the category of bimodules over a semisimple ring. Twisted tensor products of algebras provide other important examples.

We further discuss how our results on Koszul pairs can be exploited to investigate Hochschild (co)homology of Koszul rings.

Keywords: Koszul ring, Twisted tensor product, Hochschild (co)homology.

MSC 2010: 16S37, 16E40

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