SESIONES ESPECIALES

Congreso RSME 2013

S8

Análisis Complejo y Teoría de Operadores

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Near invariance and kernels of Toeplitz operators

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On Toeplizt products on Bergman spaces

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Operadores de Toeplitz y normas mixtas

Jue 24, 17:00 - 17:45, Aula 10 – A. Nicolau:
Inner functions with derivative in the weak Hardy space

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Derivada Schwarziana de funciones armónicas

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Prime and semi-prime inner functions

Vie 25, 11:45 - 12:30, Aula 10 − D. García:
Bohr's absolutely convergence problem

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Rota's Universal Operators and invariant subspaces in Hilbert spaces

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Near invariance and kernels of Toeplitz operators

M. Cristina Câmara[1], **Jonathan R. Partington**[2]

This talk presents the work of [2], a study of kernels of Toeplitz operators on scalar and vector-valued H_p spaces (for $1 < p < \infty$). The property of near invariance of a kernel for the backward shift is shown to hold in much greater generality. In the scalar case, and in some vectorial cases, the existence of a minimal kernel containing a given function is established, and a corresponding Toeplitz symbol is determined; thus for rational symbols its dimension can be easily calculated. It is shown that every Toeplitz kernel in H_p is the minimal kernel for some function lying in it. Some recent related work can be found in [1, 3, 4].

Keywords: Toeplitz operator, Toeplitz kernel, nearly-invariant subspace, model space, inner–outer factorization, Riemann–Hilbert problem

MSC 2010: 47B35, 30H10, 46E40

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On Toeplitz products on Bergman space

Alexandru Aleman[1], **Sandra Pott**[2], **Maria Carmen Reguera**[3]

In the early 90's, D. Sarason posed conjectures on the characterization of the boundedness of Toeplitz products on Hardy and Bergman spaces. The Hardy space case attracted much attention because of its close relation to the joint A_2 conjecture for the famous two-weight problem for the Hilbert transform in Real Analysis, pointed out by Cruz-Uribe in [1], but both conjectures, the Sarason conjecture for Toeplitz products on Hardy space and the joint A_2 conjecture, were shown to be false by F. Nazarov around 2000 [2].

The Bergman space case of Sarason's conjecture is still open, and is likewise connected to two-weighted inequalities on Bergman space.

In the talk, I will present a dyadic model for Toeplitz products on Bergman space, give necessary and sufficient conditions in this case, comment on necessary and sufficient conditions for the Toeplitz products, and present some counterexamples of extended versions of the Sarason Conjecture.

Keywords: Bergman space, Toeplitz operators, dyadic models

MSC 2010: Primary: 47B38, 30H20 Secondary: 42C40, 42A61,42A50

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Operadores de Toeplitz y normas mixtas

Oscar Blasco[1], **Salvador Pérez Esteva**[1]

Estudiamos clases $S(p, q)$ de operadores T en el espacio de Bergman en el disco A_2 tales que $\left(\sum_j \|T\Delta_j\|_p^q\right)^{1/q} < \infty, p, q > 0$ donde $\|\cdot\|_p$ definen las clases de Schatten en A_2, la proyección $\Delta_j f = \sum_{n\in I_j} a_n z^n$ para $f(z) = \sum_{n=0}^{\infty} a_n z^n$ y $I_j = [2^j - 1, 2^{j+1}) \cap (N \cup \{0\})$ para $j \in N \cup \{0\}$. Se verá la relación de esta propiedad con las normas mixtas de la transformada de Berezin de T y de la función asociada $f_T(z) = \|T(k_z)\|$ donde k_z es el núcleo de Bergman normalizado. En el caso de operadores de Toepltz, estas clases estan relacionadas con los llamados operadores de Schatten-Herz donde la descomposición diádica de los operadores se hace en el símbolo del operador, mediante la decomposición del disco en anillos diádicos.

Keywords: Toeplitz operators, Schatten classes, Berezin transform

MSC 2010: 47B35, 46E30

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Inner functions with derivatives in the Weak Hardy Space

Joseph A. Cima[1], **Artur Nicolau**[2]

It will be proved that exponential Blaschke products are the inner functions whose derivative is in the weak Hardy space. Exponential Blaschke products will be described in terms of their logarithmic means and in terms of the behavior of the derivatives of functions in the corresponding model space. Joint work with Joseph A. Cima

Keywords: Inner Function, Weak Hardy space, Model Space

MSC 2010: 30H10 , 30H05

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Superposition operators between Q^s-spaces and spaces of Dirichlet type

Daniel Girela

If φ is an entire function then the superposition operator S_φ is defined by

$$S_\varphi(f)(z) = \varphi\left(f(z)\right), \quad f \in \mathcal{H}ol(\mathbb{D}), \quad z \in \mathbb{D}.$$

We raise the question of characterizing the entire functions φ which transform the conformally invariant space Q^s ($0 \leq s < \infty$) into the space of Dirichlet type $\mathcal{D}^p_\alpha$ ($0 < p < \infty$, $\alpha > -1$) by superposition, and conversely. We shall pay a special attention to the case $s = 1$ and $\alpha = p - 1$, that is, we shall deal mainly with the superposition operators between $BMOA$ and the spaces $\mathcal{D}^p_{p-1}$ and compare them with those between $BMOA$ and the Hardy spaces H^p.

The content of this talk is based on work in collaboration with M. Auxiliadora Márquez.

Keywords: Superposition operators, Spaces of Dirichlet type, $BMOA$, Q^s-spaces, Besov spaces

MSC 2010: 30H25, 47H30

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Derivada Schwarziana de funciones armónicas

María J. Martín[1]

La *derivada Schwarziana* de una función analítica localmente univalente F se define como

$$S(F) = \left(\frac{F''}{F'}\right)' - \frac{1}{2}\left(\frac{F''}{F'}\right)^2 .$$

Este operador –cuya definición se atribuye a H. Schwarz $(1843 - 1921)$ aunque Kummer [6] ya lo había utilizado en 1836 en su estudio de ecuaciones diferenciales hipergeométricas– juega un importante papel en la teoría de funciones univalentes y espacios de Teichmüller.

Krauss [5] demostró que $\|S(F)\| = \sup_{z\in\mathbf{D}} |S(F)(z)|\cdot(1-|z|^2)^2 \leq 6$ para toda función analítica F que pertenece a la clase $\mathcal{S}$ de funciones holomorfas *univalentes* en el disco unidad $\mathbf{D}$ con la normalización $F(0) = F'(0) - 1 = 0$.

En 1964, Pommerenke [7] estudia las llamadas *familias linealmente invariantes* $\mathcal{F}$ y encuentra una relación entre la derivada Schwarziana y el segundo coeficiente de Taylor $a_2(F)$ de las funciones $F \in \mathcal{F}$. Concretamente, demuestra que para este tipo de familias,

$$\sup_{F\in\mathcal{F}} |a_2(F)| \leq \sqrt{1 + \frac{\sup_{F\in\mathcal{F}} \|S(F)\|}{2}} . \quad (1)$$

La clase $\mathcal{S}$ resulta ser linealmente invariante. Como consecuencia directa del teorema de Krauss y de (1), se obtiene el famoso teorema de Bieberbach que afirma que el módulo del segundo coeficiente de Taylor de toda función $F \in \mathcal{S}$ es menor o igual a 2.

En un dominio símplemente conexo Ω del plano complejo, una función armónica f puede escribirse como $f = h + \overline{g}$, donde h y g son funciones analíticas en Ω. La clase análoga a $\mathcal{S}$ en el caso armónico es la familia S_H^0 de funciones armónicas univalentes $f = h + \overline{g}$ en $\mathbf{D}$ con $h(0) = g(0) = g'(0) = 1 - h'(0) = 0$. Existe un gran número de problemas no resueltos en esta clase. Uno de los más importantes es el llamado " problema del a_2":

Es cierto que si $f = h + \overline{g} \in S_H^0$, el segundo coeficiente de Taylor $a_2(h)$ de h satisface que $|a_2(h)| < 5/2$?

Una respuesta afirmativa implicaría resultados precisos en teoremas de distorsión y recubrimiento para la clase S_H^0. La mejor cota conocida hasta el momento para $|a_2(h)|$ es 49.

Utilizando las propiedades geométricas de la superficie mínima asociada a una aplicación armónica en el disco unidad con segunda dilatación compleja $\omega = q^2$ -para cierta aplicación analítica q en $\mathbf{D}$-, Chuaqui, Duren y Osgood [3] propusieron una definición de derivada Schwarziana S_1 para este tipo de funciones armónicas. No obstante, esa condición sobre la dilatación hace que la definición no sea, en cierta forma, la más natural. Recientemente (véase [4]), hemos presentado una nueva propuesta de derivada Schwarziana S_2 definida, esta vez, para *todas* las aplicaciones armónicas localmente univalentes en el disco unidad usando un procedimiento análogo a aquel empleado por Tamanoi [8] para la derivada Schwarziana clásica.

En esta charla, mostraremos que los resultados obtenidos por Chuaqui, Duren y Osgood también son ciertos para la "nueva" derivada Schwarziana S_2. En particular, obtenemos que $M = \sup_{f\in S_H^0} ||S_2(f)||$, donde $||S_2(f)|| = \sup_{z\in\mathbf{D}} |S_2(f)(z)| \cdot (1-|z|^2)^2$, es finito. También veremos que la nueva derivada posee una importante propiedad que no es satisfecha por S_1: Su invarianza por pre-composiciones con funciones armónicas afines. Utilizando esta propiedad, generalizamos los resultados clásicos de Ahlfors [1] y Becker [2] sobre las extensiones quasiconformes de funciones analíticas. Finalmente, demostraremos que la siguiente fórmula se cumple para las funciones en S_H^0:

$$\sup_{f=h+\overline{g}\in S_H^0} |a_2(h)| \leq \sqrt{\frac{3}{2}+\frac{M}{2}}\,.$$

Este es un trabajo conjunto con los profesores M. Chuaqui y R. Hernández.

Keywords: Derivada Schwarziana, univalencia, aplicaciones armónicas, extensión quasiconforme, problema del a_2 (a_2-problem)

MSC 2010: 30C55, 30C45, 31A05

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The Cesàro operator acting on ℓ^p and consequences for Hardy spaces on the disc

Guillermo P. Curbera

The Cesàro operator on sequences, given by

$$a = (a_n)_0^\infty \in \mathbb{C}^\mathbb{N} \longmapsto \mathcal{C}(a) := \Big(\frac{1}{n+1}\sum_{k=0}^{n} a_k\Big)_{n=0}^{\infty} \in \mathbb{C}^\mathbb{N},$$

is known to be bounded on ℓ^p, for $1 < p < \infty$. From this starting point several sequence spaces arise, as

$$ces_p := \Big\{a = (a_n)_0^\infty \in \mathbb{C}^\mathbb{N} : \mathcal{C}(|a|) = \Big(\frac{1}{n+1}\sum_{k=0}^{n} |a_k|\Big)_{n=0}^{\infty} \in \ell^p\Big\},$$

which has been thoroughly studied by Bennett, [1], and

$$[\mathcal{C}, \ell^p] := \Big\{a = (a_n)_0^\infty \in \mathbb{C}^\mathbb{N} : \mathcal{C}(a) = \Big(\frac{1}{n+1}\sum_{k=0}^{n} a_k\Big)_{n=0}^{\infty} \in \ell^p\Big\},$$

which has been considered in [2] and [3]. We study the Cesàro operator on these sequence spaces, and deduce consequences for the Cesáro operator acting on the Hardy spaces on the disc:

$$f(z) = \sum_{n=0}^{\infty} a_k z^k \in H^p(\mathbb{D}) \longmapsto \mathcal{C}(f)(z) := \sum_{n=0}^{\infty} \Big(\frac{1}{n+1}\sum_{k=0}^{n} a_k\Big) z^n \in H^p(\mathbb{D}).$$

Work in collaboration with Werner J. Ricker, from the Katholische Universität Eichstätt–Ingolstadt (Germany).

Keywords: Cesáro operator, ℓ^p–spaces, Hardy spaces

MSC 2010: 30D55, 47B38

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Prime and semi-prime inner functions

Isabelle Chalendar[1], **Pamela Gorkin**[2], **Jonathan R. Partington**[3]

An inner function is a bounded analytic function on the unit disk whose radial limits have modulus one at almost every point of the unit circle. Due to classical results of A. Beurling and others, inner functions have a crucial role in the theory of Hardy spaces and the operators acting on them. The problem of when there is a non-trivia "factoring" of an inner function as the composition of other inner functions was introduced by K. Stephenson 30 years ago.

An inner function is called prime if in any such composition one of the factors is a Möbius transformation, and semiprime if a factor must be a finite Blaschke product. In this work we study when inner functions formed from various classes Blaschke products are prime or semiprime.

We show that prime finite Blaschke products are dense in the set of all finite Blaschke products and thus weak-* dense in the set of all inner functions. We also prove that finite products of thin Blaschke products can be uniformly approximated by prime Blaschke products.

Keywords: inner function, composition, prime function, semiprime function, thin Blaschke product

MSC 2010: 30D05, 30J05, 30J10, 46J20

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Bohr's absolute convergence problem

Domingo García[1]

Each Dirichlet series $D = \sum_{n=1}^{\infty} a_n \frac{1}{n^s}$, with variable $s \in \mathbf{C}$ and coefficients $a_n \in \mathbf{C}$, has a Bohr strip, the largest strip in $\mathbf{C}$ on which D converges uniformly but not absolutely. The classical Bohr-Bohnenblust-Hille Theorem states that the width of the largest possible Bohr strip equals $1/2$. Recently, this deep work of Bohr, Bohnenblust and Hille from the beginning of the last century was revisited by various authors. New methods from different fields of modern analysis allow to improve the Bohr-Bohnenblust-Hille cycle of ideas, and to extend it to new settings, in particular to Dirichlet series which coefficients in Banach spaces. In this talk we study the Bohr's absolute convergence problem: to determine the maximal width of the strip in $\mathbf{C}$ on which a Dirichlet series converges uniformly but not absolutely in a Banach space X.

Keywords: Dirichlet series, power series, polynomials, Banach spaces

MSC 2010: Primary 32A05; Secondary 46B07, 46B09, 46G20

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Rota's Universal Operators and Invariant Subspaces in Hilbert Spaces

Carl C. Cowen[1,2], **Eva A. Gallardo Gutiérrez**[2]

Rota showed, in 1960, that there are operators T that provide models for every bounded linear operator on a separable infinite dimensional Hilbert space, in the sense that given an operator A on such a Hilbert space, there is $\lambda \neq 0$ and an invariant subspace M for T such that the restriction of T to M is similar to λA. In 1969, Caradus provided a practical condition for identifying such universal operators. In this talk, we will use the Caradus theorem to exhibit a new example of a universal operator and show how it can be used to provide information about invariant subspaces for Hilbert space operators.

Keywords: Invariant subspace, composition operator, Toeplitz operator

MSC 2010: Primary: 47A15; Secondary: 47B33, 47B35.

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