SESIONES ESPECIALES

Congreso RSME 2013

S9
Singularidades

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On equivariant analogues of Poincaré series and monodromy zeta functions

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The Euler characteristic of an algebraic variety, its counting polynomial and its Chern-Schwartz-MacPherson class

Jue 24, 12:00 - 12:25, Aula 8 – Helena Cobo Palacios:
Jet schemes of quasi-ordinary surface singularities

Jue 24, 12:30 - 12:55, Aula 8 – Hussein Mourtada:
Jet schemes and embedded resolutions of singularities

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The Mond number of a map germ from the plane to 3-space

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Clasificación topológica de gérmenes de aplicación de corrango 2 de $\mathbb{R}^2$ en $\mathbb{R}^2$

Jue 24, 18:00 - 18:25, Aula 8 – María Cruz Fernández Fernández:
Irregular singularities of modified A-hypergeometric systems

Jue 24, 18:30 - 18:55, Aula 8 – Carlos Galindo Pastor:
Números Jumping e Ideales Multiplicadores de ideales completos de anillos locales regulares bidimensionales

Jue 24, 19:00 - 19:25, Aula 8 – Pierrette Cassou-Noguès:
On simple and quasi-simple rational polynomials

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Santiago de Compostela, 21–25 enero 2013

On equivariant analogues of Poincaré series and monodromy zeta functions

S. Gusein-Zade[1]

A number of invariants of singularities of analytic spaces and of germ of analytic functions on them have equivariant versions for singularities with an action of a finite group G and for G-invariant (or G-equivariant) function germs on them. Equivariant analogues of invariants could both be used for study of equivariant situations and to clarify properties of the usual, non-equivariant invariants. Somewhat mysterious are relations between the Poincaré series of natural filtrations and monodromy zeta functions. Equivariant versions of the Poincaré series and of the monodromy zeta functions are far from being straightforward. We discuss the use of Burnside rings and their analogues for construction of such equivariant versions and some results obtained in this direction.

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The Euler characteristic of an algebraic variety, its counting polynomial and its Chern-Schwartz-MacPherson class.

Miguel Angel Marco Buzunariz[1]

The talk presents a method to compute the Euler characteristic of a complex algebraic variety. This method consists in projecting to a linear space, and comparing the Euler characteristic of the variety to the one of the linear space. This idea generalizes in a natural way to a richer invariant, which coincides with the counting polynomial and is directly related to the Chern-Schwartz-MacPherson class. Finally, we show how this invariant can be computed by counting points of varieties over finite fields.

MSC 2010: 14F45, 1404

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Jet schemes of quasi-ordinary surface singularities

Helena Cobo[1]

Jet schemes have attracted a lot of attention thanks to their role in Motivic Integration and applications to Birational Geometry. But already in simple cases their geometry is very complicated ([3],[4]). In this talk I will report on recent results where we determine the geometric structure of the jet schemes of quasi-ordinary surface singularities. This is a joint work with Hussein Mourtada.

Keywords: Jet schemes, quasi-ordinary singularities

MSC 2010: 14E18, 14B05

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Jet schemes and embedded resolutions of singularities

Hussein Mourtada

I will talk about the relation between the irreducible components of jet schemes of an algebraic variety and its embedded resolution of singularities. I mainly will report about results for plane curves and examples of surfaces. Part of these results are joint work with Monique Lejeune-Jalabert and Ana Reguera.

Keywords: Jet schemes, resolution of singularities

MSC 2010: 14E18,14B05

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Sobre el tipo topológico del discriminante

F. Delgado[1], **H. Maugendre**[2]

Dado un morfismo finito del plano complejo en sí mismo, el tipo topológico de su discriminante depende del tipo analítico de las funciones componentes y no sólo de su tipo topológico. Como punto de partida se describe la topología de la imagen de un germen irreducible cualquiera mediante el uso iterado de pencils construidos de forma natural a partir de las funciones componentes y del germen de partida. La relación conocida entre las ramas del lugar crítico de la aplicación y las fibras especiales del pencil definido por sus funciones componente permite determinar el tipo topológico de la curva discriminante. Los resultados han sido obtenidos en colaboración con H. Maugendre (Institut Fourier, Grenoble).

Keywords: discriminant, topological type, iterated pencils, analytic morphisms

MSC 2010: 14H20, 32S05, 32S15, 32S45, 32S55

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The Mond number of a map germ from the plane to 3-space

Ton Marar[1]

Finite determinacy of germs of functions $f : (C^n, 0) \to (C, 0)$ is equivalent to the finiteness of the Milnor number $\mu(f)$, that is, the number of Morse points in a generic deformation of f.

For map germs $f : (C^2, 0) \to (C^3, 0)$ the corresponding number of non degenerate critical points is $C(f)$, the number of Whitney umbrellas in a generic deformation of f and it is given by the vanishing of the 2×2 minors of the Jacobian matrix of f.

David Mond [2] notice that the finiteness of $C(f)$ is necessary but not sufficient for finite determinacy of the map germ. Thus, he introduced two other invariants whose finiteness together with that of $C(f)$ is equivalent to finite determinacy of the map germ.

We prove that all the three of Mond's invariants compose themselves as a linear combination into a single analytic invariant and we call it the *M*ond number of $f : (C^2, 0) \to (C^3, 0)$.

Examples of map germs of corank 1 and corank 2 from [1] will be presented.

Keywords: map germs, double points, finite determinacy

MSC 2010: 32S15

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Clasificación topológica de gérmenes de aplicación de corrango 2 de $\mathbb{R}^2$ en $\mathbb{R}^2$

Juan Antonio Moya Pérez[1], **Juan José Nuño Ballesteros**[1]

Después de haber definido un invariante topológico completo para gérmenes finitamente determinados $f : (\mathbb{R}^2, 0) \to (\mathbb{R}^2, 0)$, usándolo para clasificar gérmenes de este tipo en el caso de corrango 1 en [6], en este trabajo abordamos la difícil tarea de extender esta clasificación a corrango 2. Concretamente, nos centraremos en los pertenecientes a la clase $\Sigma^{2,0}$.

Keywords: Gauss word, link, finite determinacy

MSC 2010: 58K15, 58K40

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Irregular singularities of modified A-hypergeometric systems

F. J. Castro Jiménez[1], **M. C. Fernández Fernández**[1], **T. Koike**[2], **N. Takayama**[2]

This talk is based on the paper [1]. A modified A-hypergeometric system [6] is a system of linear partial differential equations in the complex variables $x_1, \ldots, x_n, t$ for the function $f(t^{w_1}x_1, \ldots, t^{w_n}x_n)$ where $f(y_1, \ldots, y_n)$ is a holomorphic solution of an A-hypergeometric system (or GKZ-system after [4]) and w is a weight vector with integer coordinates. We study the irregularity of modified systems by adapting to this case the notion of umbrella introduced in [5]. Especially, we study their slopes and their Gevrey series solutions. We also give Laplace integral representations of divergent series solutions of the modified system under some conditions on the weight vector. As application of this study, we interpret the Gevrey series solutions of the original A-hypergeometric system along coordinate varieties, as constructed in [2] and [3], as asymptotic expansions of hypergeometric functions along toric curves.

Keywords: hypergeometric system, Gevrey series, slopes, asymptotic expansion

MSC 2010: 33C70, 13N10

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Números Jumping e Ideales Multiplicadores de ideales completos de anillos locales regulares bidimensionales

Carlos Galindo[1]

Los ideales multiplicadores son una herramienta reciente e importante de la teoría de singularidades y de la geometría birracional. Tienen la ventaja de que proporcionan información sobre el tipo de singularidad del ideal, divisor o métrica al que se asocian y satisfacen varios teoremas de anulación, lo que los hace muy útiles. La familia de ideales multiplicadores asociada a (digamos) un ideal está totalmente ordenada por inclusión y viene parametrizada por una familia de números racionales no negativos, denominados números jumping. A pesar de la utilidad de estos ideales y números, se conocen explícitamente en muy pocos casos debido a que su cálculo, en casos genéricos, es complicado.

En la charla, consideraremos un anillo local regular bidimensional R sobre el cuerpo complejo y un ideal completo I de R. Cuando I es simple, recordaremos quién es su familia de números jumping [3] e introduciremos la serie de Poincaré de ideales multiplicadores de I. Éste es un objeto algebraico que agrupa números jumping e ideales multiplicadores y, para él, mostraremos una expresión explícita y muy simple. En el caso en que I no es simple, daremos una fórmula explícita que permite calcular el menor de los números jumping, llamado umbral log-canónico. Esta fórmula se deduce fácilmente de otra que hemos obtenido para el cálculo del citado umbral para gérmenes reducidos de curva plana. Fórmula que es válida en cualquier cuerpo y se expresa en términos de los valores de contacto maximal de las ramas del germen y de la multiplicidad de intersección entre ellas.

Los resultados aquí expuestos han sido obtenidos junto a F. Monserrat y, en el caso del umbral log-canónico, también junto a F. Hernando. Pueden ser consultados en [1, 2].

Keywords: Ideales Multiplicadores, Números Jumping, Umbral Log-canónico.

MSC 2010: 14B05, 13H05.

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On simple and quasi-simple rational polynomials

Pierrette Cassou-Noguès[1], **Daniel Daigle**[2]

In this talk we will explain an elementary combinatorial proof of the classification of rational polynomials of simple type due to Neumann and Norbury. We will show how the method can be used to deal with quasi-simple rational polynomials and recover results of Sasao.

Keywords: Affine Geometry, rational polynomials

MSC 2010: 14H20, 32S50

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