SESIONES ESPECIALES

Congreso RSME 2013

S11
Harmonic Analysis

Mie 23, 17:00 - 17:45, Aula 5 – Javier Duoandikoetxea:
Weights for the Calderón operator and characterizations of A_∞

Mie 23, 17:50 - 18:35, Aula 5 – Francisco Javier Martin-Reyes:
Convergence of lacunary ergodic Cesàro averages and weighted inequalities

Mie 23, 18:40 - 19:35, Aula 5 – Luigi Fontana:
Exponential Integrability: classical results, recent developments and some open problems

Jue 24, 11:00 - 11:45, Aula 5 – Andrei K. Lerner:
Sharp weighted bounds for multilinear maximal functions and Calderón-Zygmund operators

Jue 24, 11:50 - 12:35, Aula 5 – Kabe Moen:
Improved weighted estimates for classical operators

Jue 24, 12:40 - 13:35, Aula 5 – Ioannis Parissis:
The endpoint Fefferman-Stein inequality for the strong maximal function

Jue 24, 17:00 - 17:45, Aula 5 – Jorge J. Betancor:
Vector valued multivariate spectral multipliers, Littlewood-Paley functions, and Sobolev spaces in the Hermite setting

Jue 24, 17:50 - 18:35, Aula 5 – Guixiang Hong:
Calderón-Zygmund operators associated to matrix-valued kernels

Jue 24, 18:40 - 19:35, Aula 5 – Ezequiel Rela:
Sharp norm inequalities with A_∞ weights for the Hardy-Littlewood maximal function on Spaces of Homogeneous Type

Vie 25, 11:00 - 11:45, Aula 5 – Joan Mateu:
L^p estimates for the maximal singular integral in terms of the singular integral.

Vie 25, 11:50 - 12:35, Aula 5 – F.J. Pérez Lázaro:
On Gagliardo-Nirenberg type inequalities

Vie 25, 12:40 - 13:35, Aula 5 – Jesús Munárriz Aldaz:
The centered Hardy-Littlewood maximal operator in high dimensions

Vie 25, 17:00 - 17:45, Aula 5 – José L. Torrea:
Harnack's inequality for fractional operators

Vie 25, 17:50 - 18:35, Aula 5 – María Carmen Reguera:
(TBA)

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WEIGHTS FOR THE CALDERÓN OPERATOR AND CHARACTERIZATIONS OF A_∞

Javier Duoandikoetxea[1], **Francisco J. Martín-Reyes**[2], **Sheldy Ombrosi**[3]

The Calderón operator is the sum of the Hardy averaging operator and its adjoint. We show that the weights for which the Calderón operator is bounded on $L^p(w)$ or is of weak-type (p,p) with respect to $w(x)dx$ are the same as those corresponding to the Muckenhoupt basis in $(0,\infty)$ formed by intervals starting at 0. The weights for the strong type and the weak type coincide for $1 < p < \infty$ and differ for $p = 1$. We prove that the same classes of weights are obtained considering the sum of the Riemann-Liouville and Weyl averaging operators. Several counterexamples are given to show that some of the typical properties of the usual A_p weights, like the reverse Hölder inequality, can fail for this class of weights.

In the second part of the talk we discuss the different characterizations of A_∞ in the case of general bases. In particular, we show that most of them define different classes of weights for the above-mentioned basis related to the Calderón operator.

Keywords: Calderón operator, weighted inequalities, Muckenhoupt bases, A_∞ classes

MSC 2010: 42B25, 47G10

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Convergence of lacunary ergodic Cesàro averages and weighted inequalities

Francisco Javier Martín Reyes [1]

Let T be a positive linear operator with positive inverse. We consider the ergodic Cesàro-α averages

$$\mathcal{A}_{n,\alpha} f = \frac{1}{A_n^\alpha} \sum_{k=0}^{n} A_{n-k}^{\alpha-1} T^k f, \quad 0 < \alpha \leq 1,$$

and the ergodic Cesàro-α maximal operator associated to T. For Lebesgue spaces $L^p(\nu)$, the good range for the convergence of the Cesàro-α averages and the boundedness of the maximal operator is $1/\alpha < p < +\infty$. In this lecture we shall recall previous results about convergence of ergodic averages and we shall present some of the results in [1] about the convergence of the lacunary sequence $\mathcal{A}_{2^k,\alpha} f$ and the boundedness of its associated ergodic maximal operator. We get positive results in the range $1 \leq p < \frac{1}{1-\alpha}$. We use transference arguments which leads to us to study in depth weighted inequalities of the lacunary Cesàro-α maximal operator in the setting of the integers and in the setting of the real line.

Keywords: Cesàro-α ergodic averages, lacunary ergodic averages, ergodic maximal operator, positive operator, nonsingular transformation, weighted inequalities

MSC 2010: 47A35, 37A40, 42B25

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Exponential Integrability: classical results, recent developments and some open problems

Luigi Fontana [1]

As early as 1929 Zygmund proved sharp exponential integrability for the conjugate function on L^∞ on the unit circle. However, after the works of Trudinger (1967), Moser (1971) and Adams (1988), exponential integrability is commonly associated with the critical cases of Sobolev embeddings. In this lecture, after a survey of the classical results, we will present some recent developments focusing particularly on joint work of C. Morpurgo and the speaker, give a glimpse of the proofs and mention some open problems

Keywords: Sobolev embeddings, Moser-Trudinger inequalities, Exponential integrability

MSC 2010: 46E35, 26D10

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Sharp weighted bounds for multilinear maximal functions and Calderón-Zygmund operators

Andrei K. Lerner [1]

Recently sharp weighted A_p and mixed $A_p - A_\infty$ estimates have been obtained for the Hardy-Littlewood and Calderón-Zygmund operators. We shall discuss some extensions of these results to a multilinear setting. In particular, we obtain a multilinear version of the A_2 conjecture. Based on joint work with W. Damián and C. Pérez.

Keywords: Multilinear maximal operator, Calderón-Zygmund theory, sharp weighted bounds.

MSC 2010: 42B20, 42B25

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Improved weighted estimates for classical operators

Kabe Moen[1]

We will talk about some recent advances concerning the A_2 theorem. First we give a short proof of the sharp bound for singular integrals that holds for all p, $1 < p < \infty$ from [4]. The novelty of this proof is that it avoids extrapolation and two weight testing conditions. We will also talk about some mixed estimates for singular and fractional integrals where the constants only involve one supremum. This talk will be based on joint works with A. K. Lerner [3] and D. Cruz-Uribe [1, 2].

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The endpoint Fefferman-Stein inequality for the strong maximal function

Teresa Luque[1], **Ioannis Parissis**[2]

Let Mf denote the strong maximal function of f, that is the maximal average of f with respect to rectangles parallel to the coordinate axes. We prove the natural endpoint Fefferman-Stein inequality for M and any strong Muckenhoupt weight w:

$$w(\{x \in \mathbf{R}^n : Mf(x) > \lambda\}) \leq C_{w,n} \int_{\mathbf{R}^n} \frac{|f(x)|}{\lambda}\Big(1+\big(\log^+ \frac{|f(x)|}{\lambda}\big)^{n-1}\Big) Mw(x)dx.$$

This is extends the corresponding two-dimensional result of T. Mitsis.

Keywords: Fefferman-Stein, rectangles, strong maximal function

MSC 2010: 42B25, 42B2

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Vector valued multivariate spectral multipliers, Littlewood-Paley functions, and Sobolev spaces in the Hermite setting

Jorge. J. Betancor [1], **Juan C. Farina**[1], **A. Sanabria**[1]

In this paper we find new equivalent norms in $L^p(\mathbb{R}^n, \mathbb{B})$ by using multivariable Littlewood-Paley functions associated with Poisson semigroup for the Hermite operator, provided that $\mathbb{B}$ is a UMD Banach space with the property (α). In order to do this we previously establish the corresponding property when the classical Poisson semigroup is considered and we use γ-radonifying operator. Our new equivalent norms allow us to obtain $L^p(\mathbb{R}^n, \mathbb{B})$-boundedness properties for (vector valued) multivariable spectral multipliers for Hermite operators. As application of this Hermite multiplier theorem we prove that the Banach valued Hermite Sobolev and potential spaces coincide.

Keywords: spectral multipliers, Hermite, Sobolev spaces

MSC 2010: 42C05, 42C15

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Calderón-Zygmund operators associated to matrix-valued kernels

Guixiang Hong[1]

Calderón-Zygmund operators with noncommuting kernels may fail to be L_p-bounded for $p \neq 2$, even for kernels with good size and smoothness properties. Matrix-valued paraproducts, Fourier multipliers on group vNa's or noncommutative martingale transforms are frameworks where we find such difficulties. We obtain weak type estimates for perfect dyadic CZO's and cancellative Haar shifts associated to noncommuting kernels in terms of a row/column decomposition of the function. Arbitrary CZO's satisfy $\mathrm{H}_1 \to L_1$ type estimates. In conjunction with $L_\infty \to \mathrm{BMO}$, we get certain row/column L_p estimates. Our approach also applies to noncommutative paraproducts or martingale transforms with noncommuting symbols/coefficients. Our results complement recent results of Junge, Mei, Parcet and Randrianantoanina.

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Sharp norm inequalities with A_∞ weights for the Hardy–Littlewood maximal function on Spaces of Homogeneous Type

Ezequiel Rela[1]

We present quantitative versions of one-weight and two-weight theorems for the Hardy-Littlewood maximal operator within the context of spaces of homogeneous type.

One way to obtain such results is by using a sharp Reverse Hölder Inequality for A_∞ weights. Although it is not strictly necessary, this inequality is interesting on its own and therefore we present here a new and recent proof, with the extra advantage of allowing us to extend the result to any space of homogeneous type. In this case, we obtain a *weak* RHI which is still sharp in the dependence on the A_∞ constant of the weight. Then we can derive a precise version of the classical open property for A_p classes and the L^p boundedness of the maximal function.

We also will show a more general result concerning the problem of finding sufficient conditions for the two-weight boundedness of the H–L maximal operator. We provide sufficient conditions in terms of Orlicz norms and obtain, as a corollary, a different and direct proof of the sharp one-weight inequality *without* using the Reverse Hölder Inequality.

Keywords: Space of homogeneous type, Muckenhoupt weights, Reverse Hölder, Maximal functions, Sawyer's theorem

MSC 2010: 42B25, 43A85

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L^p estimates for the maximal singular integral in terms of the singular integral

Joan Mateu[1]

In this talk we describe a complete characterization of Calderón-Zygmund operators for which one can control the weighted L^p norm of the maximal singular operator by the weighted L^P norm of the operator for any weight in the Muckenhoupt class, $1 < p < \infty$.

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On Gagliardo-Nirenberg type inequalities

V.I. Kolyada[1], **F.J. Pérez Lázaro**[2]

We present a Gagliardo-Nirenberg inequality which bounds Lorentz norms of the function by Sobolev norms and homogeneous Besov quasinorms with negative smoothness. We prove also other versions involving Besov or Triebel-Lizorkin quasinorms. These inequalities can be considered as refinements of Sobolev type embeddings. They can also be applied to obtain Gagliardo-Nirenberg inequalities in some limiting cases. Our work extends results of [1, 2, 3, 4, 5, 6, 7] in some sense. Our methods are based on estimates of rearrangements in terms of heat kernels. These methods enable us to cover also the case of Sobolev norms with $p = 1$.

Keywords: Gagliardo-Nirenberg inequality, Sobolev spaces, Besov spaces, Triebel-Lizorkin spaces

MSC 2010: Primary 46E35, 26D10; Secondary 46E30

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The centered Hardy-Littlewood maximal operator in high dimensions

Jesús Munárriz Aldaz[1]

We discuss some recent and not so recent results by several authors, regarding the centered Hardy-Littlewood maximal operator M, and more specifically, how the best constants in the weak and strong type inequalities change with the dimension. Open problems will also be mentioned.

Keywords: Hardy-Littlewood maximal operator

MSC 2010: 42B25

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Harnack's inequality for fractional operators

José L. Torrea

We shall discuss interior Harnack's inequalities for solutions of fractional non-local equations. Our examples include fractional powers of divergence form elliptic operators with potentials and operators arising in classical orthogonal expansions. To get the results we use Semigroup theory, a generalization of the Caffarelli-Silvestre extension problem, the Harnack's inequality for degenerate Schrödinger operators proved by C. E. Gutiérrez, and a transference method. In this manner we apply local PDE techniques to nonlocal operators.

Keywords: Semigroups, Fractional powers of operators, Harnack's inequality.

MSC 2010: 35R11, 47D06

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