"POSTERS"

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A simulated annealing algorithm for vehicle routing optimization: a graphical interface in Java

Roque M. Guitián de Frutos[1], **Balbina V. Casas Méndez**[2]

This work considers the problem of minimizing the cost of carrying out the routes of the vehicles used by a real agricultural cooperative that distributes animal feed among the partners. Because solving the exact model is computationally burdensome, we propose to implement heuristic algorithms to reduce the computational time. To this end, first an initial solution is obtained through an insertion heuristic algorithm. The algorithm works by building paths successively until all orders of the partners are planned. Secondly, we design a simulated annealing algorithm that works on the initial solution for possible improvements. One difficulty with this approach lies in the way of obtaining neighboring solutions with the objective of reaching optimal solutions. In order to implement an efficient software, seven ways have been designed and programmed to get neighboring solutions randomly interspersed. The execution time and the maximum iterations must be specified by the user according to their needs. Finally, we built a graphical interface. With it, the user can interact with the system quickly and easily. We show the peculiarities of the interface that has been created and some of the numerical results obtained with it.

Keywords: Vehicle route optimization, simulated annealing algorithm, Java graphical interface

Mathematics Subject Classification 2010: 90C59, 90C90

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[1]Departamento de Aplicaciones Informáticas
Complejo Hospitalario Universitario de Santiago de Compostela (CHUS)
Travesía da Choupana s/n, 15706 Santiago de Compostela
roqueguitian@hotmail.com

[2]Departamento de Estadística e Investigación Operativa
Facultad de Matemáticas. Universidad de Santiago de Compostela
Rúa Lope Gómez de Marzoa s/n, Campus Sur, 15782 Santiago de Compostela
balbina.casas.mendez@usc.es

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Applications of estimates of C_0-semigroups in partial differential equations

Luciano Abadias[1], **Pedro J. Miana**[1]

An application of differential inequalities is to provide stability to the solutions of differential equations. A. Córdoba and D. Córdoba have used pointwise inequalities, satisfied by fractional derivatives, to obtain L^p-decay of solutions of the following partial differential equation,

$$(\frac{\partial}{\partial t} + u \cdot \nabla) f = -k(-\Delta)^\alpha f, \quad \text{for } 0 \leq \alpha \leq 1,$$

which are related to quasigeostrophic equation, see [1]. In this poster, we extend these pointwise inequalities for fractional powers of generators of convolution C_0-semigroups, and we obtain L^p-decay of solutions of differential equations,

$$\|f(\cdot,t)\|_p^p \leq \frac{\|f(\cdot,0)\|_p^p}{(1+\varepsilon C t \|f(\cdot,0)\|_p^{p\varepsilon})^{\frac{1}{\varepsilon}}},$$

applying Nash-type inequalities.

Keywords: Convolution C_0-semigroup, fractional power, L^p-decay

Mathematics Subject Classification 2010: 47D06, 39B62, 35B35, 35R11

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[1]Departamento de Matemáticas
IUMA & Facultad de Ciencias
Universidad de Zaragoza
Pedro Cerbuna, 12, 50009, Zaragoza, SPAIN
labadias@unizar.es
pjmiana@unizar.es

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Continuous right inverses for the asymptotic Borel map in ultraholomorphic classes via a Laplace-type transform

Alberto Lastra[1], **Stéphane Malek**[2], **Javier Sanz**[3]

A new construction of linear continuous right inverses for the asymptotic Borel map is provided in the framework of general Carleman ultraholomorphic classes in narrow sectors. Such operators were already obtained by V. Thilliez [5] by means of Whitney extension results for non quasianalytic Whitney ultradifferentiable jets on compact sets with Lipschitz boundary, due to J. Chaumat and A. M. Chollet [2], but our approach is completely different, resting on the introduction of a suitable truncated Laplace-type transform. This technique is better suited for a generalization of these results to the several variables setting, which was already obtained by the first and third authors [3] with different, and more intrincated, techniques. Moreover, this new approach closely resembles the classical procedure in the case of Gevrey classes, so indicating the way for the introduction of a new concept of summability which generalizes $k-$summability theory as developed by J. P. Ramis [4, 1].

Keywords: Laplace transform, formal power series, asymptotic expansions, ultraholomorphic classes, Borel map, extension operators

Mathematics Subject Classification 2000: 30D60, 40C10, 41A60

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[1]Departamento de Matemáticas
Universidad de Alcalá
Edificio de Ciencias, Campus universitario, 28871 Alcalá de Henares, Madrid
alberto.lastra@uah.es

[2]UFR de Mathématiques Pures et Appliquées
Université de Lille (France)
Cité Scientifique, Bât. M2, 59655 Villeneuve d'Ascq Cedex, France
malek@math.univ-lille1.fr

[3]Departamento de Álgebra, Análisis Matemático, Geometría y Topología, IMUVA
Universidad de Valladolid
Facultad de Ciencias, Paseo de Belén 7, 47011, Valladolid
jsanzg@am.uva.es

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Dominación de espacios topológicos por espacios métricos

David Guerrero Sánchez[1]

Comenzaremos la presente comunicación por observar que cualquier espacio topológico X está dominado por algún espacio métrico M. Por lo tanto, al tomar el espacio métrico de peso mínimo que domina a un espacio topológico dado, es posible presentar el siguente invariante cardinal de los espacios topológicos: el índice dm de de dominación por un métrico M de un espacio dado X. Denotamos a este invariante como $dm(X)$ para cada espacio X. Mostraremos que en general $dm(X) \leq \ell\Sigma(X)$ donde $\ell\Sigma(X)$ es el índice de K-determinación del espacio X definido en [CMO] por Cascales, Muñoz y Orihuela. Sin embargo, probaremos que ambos invariantes cardinales coinciden en la clase de los espacios angélicos. Esto nos permitirá demostrar (en ZFC) que para un espacio compacto K el espacio $C_p(K)$ es fuertemente dominado por un espacio segundo numerable si y sólo si K es numerable. Esto resuelve el problema abierto [COT, Problem 4.11] planteado por Cascales, Orihuela y Tkahcuk.

Keywords: Dominación, Dominación fuerte, espacio numerablemnte determinado, espacio Lindelöf Σ, metrizabilidad, cubrimiento compacto, espacio de funciones, espacio cósmico, $\aleph_0$-espacio

Mathematics Subject Classification 2010: 54B10, 54C05, 54D30

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[1]Departamento de Matemáticas, Universidad de Murcia, 30100 Murcia, Spain
david.guerrero@um.es

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Dynamics of entire functions universal under similarities

J.A. Prado-Bassas[1], **M.C. Calderón-moreno**[1]

In this poster we characterize the dynamics of entire functions universal under similarities and bounded on "large" sets, in terms of topological properties of these sets. Moreover, if we consider the set of universal entire functions via similarities that are bounded on sub-Arakelian sets (that is, sets which can be completed as Arakelian's), then it is shown that its algebraic size is as large as possible. As a consequence, we show several results as, for example, the existence of an algebraically large set of universal entire functions bounded on every (straight) line.

Keywords: Universal functions, sub-Arakelian subsets, similarities, dense-lineability, spaceability

Mathematics Subject Classification 2010: 30E10, 30D15, 47A1, 47B38

[1]Dpto. Análisis Matemático
Universidad de Sevilla
Avda. Reina Mercedes s/n, 41012 Sevilla SPAIN
bassas@us.es, mccm@us.es

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El método de Bases Reducidas aplicado a un problema de convección de Rayleigh-Bénard.

Francisco Pla[1], **Yvon Maday**[2], **Henar Herrero**[1]

El método de bases reducidas es un método de discretización numérico idóneo para resolver problemas que dependen de parámetros $\mathcal{P}(\phi(\mu);\mu) = 0$ con parámetro μ y juega un papel importante en diversas áreas, de la física, ingeniería o de la mecánica dado que permite acelerar los cálculos computacionales.

Este método consiste en aproximar la solución $\phi(\mu)$ de un problema $\mathcal{P}(\phi(\mu);\mu) = 0$, mediante una combinación lineal de soluciones *apropiadas* ya calculadas $\phi(\mu_i)$ tal que $\mu_i,\ i = 1, 2, ..., N$ son parámetros obtenidos mediante un proceso iterativo de minimización de medidas de *Kolmogorov n-with* [1, 2].

En este trabajo, se aplica el método de bases reducidas a un problema de convección de tipo Rayleigh-Bénard en una celda dos dimensional (2D) con viscosidad constante $\mathcal{P}(\phi(R);R) = \vec{0}$, con parámetro el número de Rayleigh R, en el que se consideran las siguientes ecuaciones adimensionalizadas incompresibles de Bousinesq Navier-Stokes con la ecuación de calor,

$$
\begin{aligned}
0 &= \nabla \cdot \vec{v}, \\
\frac{1}{Pr}\left(\partial_t \vec{v} + \vec{v}\cdot\nabla\vec{v}\right) &= R\,\theta\vec{e_3} - \nabla P + \nu\Delta\vec{v}, \\
\partial_t\theta + \vec{v}\cdot\nabla\theta &= \Delta\theta.
\end{aligned}
$$

Donde $\phi = (\vec{v},\ \theta,\ P)$ con $\vec{v} = (v_x,\ v_z)$ el campo de velocidades, θ el campo de temperatura y P la presión y los números adimensionales de Rayleigh (R) y de Prandtl (P_r).

Para cada relación de aspecto se presentan múltiples soluciones estables para diferentes números de Rayleigh que coexisten para mismos valores de parámetros geofísicos [3]. El objetivo es mediante los elementos de las bases reducidas $\{\phi(R_i), i = 1, 2, ..., N\}$ de nuestro problema en una relación de aspecto fija calcular una aproximación de solución estacionaria $\phi(R)$ en ramas estables al igual que una aproximación de la solución en los puntos de bifurcación para números de Rayleigh críticos, $\phi(R_c)$.

El problema se resuelve numéricamente mediante el método variacional de Galerkin usando las cuadraturas de Legendre Gauss-Lobatto junto con el método de bases reducidas para obtener las soluciones $\phi(R_i)$ tales que $\phi(R) \sim \sum_{i=1}^{N} \lambda_i \phi(R_i)$.

Keywords: EPD, Mecánica de Fluidos, Métodos Numéricos, Bifurcaciones

Mathematics Subject Classification 2010: 76D05, 65N30, 70K50

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[1]Departamento de Matemáticas, Facultad de Ciencias y Tecnologías Químicas
Universidad de Castilla-La Mancha
Avda. Camilo José Cela s/n, 13071 Ciudad Real, Spain
Francisco.Pla@uclm.es, Henar.Herrero@uclm.es

[2]Laboratoire Jacques-Louis Lions
Université Pierre et Marie Curie, Paris 6
F-75005, Paris, France
maday@ann.jussieu.fr

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Extreme points and the geometry of integral polynomials

Ricardo García[1]

The study of the extreme points, often in dual spaces, and the presence of M-ideal structures proved to be very useful tools in the theory of Banach spaces, leading to a better understanding of the geometry of the involved spaces as can be seen in [1, 3, 4].

Here we report on the research of [2], where we showed that the set of extreme points of the unit ball of the space of k-homogeneous integral polynomials over a real Banach space X is exactly

$$\{\pm\phi^k : \phi \in X^*, \|\phi\| = 1\}.$$

Apart from normalization, no condition on ϕ is needed. This contrasts with the "non-symmetric" situation which is much less favorable. Indeed, the extreme points of the unit ball of the space of integral k-lineal functionals are all of the form $\phi_1\phi_2\cdots\phi_k$, where each ϕ_i has to be an extreme point of the ball of X^* as can be seen in [4, 1].

Armed with this result we show that, if X is a subspace of the real Banach space Y and if X is a non trivial M-ideal in Y, then the k-th symmetric injective tensor product of Y is *never* an M-ideal in the corresponding tensor product of Y unless $k = 1$, of course.

As before, this result marks up a difference with the behavior of the "full" tensor product since, when X is an M-ideal in Y, the k-th injective tensor product of X is an M-ideal in the corresponding tensor of Y([3]).

Despite of these facts, we manage to show that if X is an Asplund space, then every integral k-homogeneous polynomial on X has a unique extension to Y that preserves the integral norm.

Keywords: Extreme point, Integral polynomial, M-ideal.

Mathematics Subject Classification 2010: 46G25, 46B28.

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[1]Departamento de Matemáticas
Universidad de Extremadura
Avenida de Elvas, (ES-06071) Badajoz, Spain
rgarcia@unex.es

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Matrix Sylvester differential equations in the theory of vector orthogonal polynomials

A. Branquinho[1], **A. Foulquié-Moreno**[2], **A. Mendes**[3]

We study the so called classical Hahn sequences of vector orthogonal polynomials, i.e., the sequences of vector orthogonal polynomials whose derivative sequence is also orthogonal.

In this work we find sequences of matrix polynomials whose orthogonality measure, W, satisfy a matrix Sylvester differential equation.

We characterize a full Kostat-Toda system in terms of a system of matrix orthogonal polynomials with a matrix measure defined as a solution of a Sylvester matrix differential equation.

Keywords: Matrix orthogonal polynomials, linear functional, recurrence relation, operator theory, matrix Sylvester differential equations, full Konstant-Toda systems.

Mathematics Subject Classification 2010: 33C45, 39B42, 15B57, 47N20, 34K99, 42C05.

[1]CMUC, Departmento de Matemática
Universidade de Coimbra
Largo D. Dinis, 3001-454 Coimbra, Portugal.
ajplb@mat.uc.pt

[2]CIDMA, Departamento de Matemática, Universidade de Aveiro, Campus de Santiago 3810, Aveiro, Portugal.
foulquie@ua.pt

[3]Departamento de Matemática
Escola Superior de Tecnologia e Gestão
Instituto Politécnico de Leiria, 2411 - 901 Leiria - Portugal.
aimendes@ipleiria.pt

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Métodos iterativos tipo Steffensen libres de Inversos

M. J. Rubio[1]

Sea una ecuación no lineal $F(x) = 0$, definida en espacios de Banach. Para aproximar numéricamente una solución consideramos métodos de tipo Steffensen ([1], [2]) que son procesos definidos mediante diferencias divididas de primer orden [5]. Estos procesos iterativos poseen la característica de no utilizar derivadas en su implementación, sin embargo, en su algoritmo sí aparecen operadores inversos, en concreto el operador inverso de una diferencia dividida de primer orden lo que produce ciertas dificultades en su aplicación. En este trabajo consideramos una modificación del método de Moser [3] aplicado a métodos tipo Steffensen para poder evitar el cálculo de dichos inversos. Para ello utilizamos una técnica basada en relaciones de recurrencia [4].

Keywords: Ecuaciones no lineales en espacios de Banach, Método de Steffensen, relaciones de recurrencia

Mathematics Subject Classification 2010: 45G10, 47H17, 65J15

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[1]Departamento de Matemáticas y Computación
Universidad de La Rioja
C/ Luis de Ulloa s/n 26004 Logroño
mjesus.rubio@dmc.unirioja.es

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Multivariate orthogonal polynomials of Sobolev type: an example on the ball.

Antonia M. Delgado[1], **Teresa E. Pérez**[1], **Miguel A. Piñar**[1]

In this presentation, we consider the Sobolev–type inner product of the form

$$(p, q)_S = (p, q)_\mu + \lambda \sum_{k=0}^{N} \frac{\partial p(s_k)}{\partial n} \frac{\partial q(s_k)}{\partial n}, \quad \lambda > 0,$$

$(\cdot, \cdot)_\mu$ is the usual inner product on the unit ball B^d, and $\frac{\partial}{\partial n}$ represents the normal derivative on the sphere S^{d-1}. Then, multivariate orthogonal polynomials of Sobolev–type and the kernel functions associated with this Sobolev–type inner product are studied. More explicitly, we express them in terms of those corresponding with the original inner product. These results are applied to obtain the asymptotics of the Christoffel functions. Finally, the special case of the SobolevÐtype modification of the bivariate classical measure on the unit disk obtained by adding the outward normal derivatives on a finite set of uniformly distributed points on the unit circle is presented.

Keywords: Multivariate orthogonal polynomials, Sobolev inner products, Christoffel functions, Asymptotics.

Mathematics Subject Classification 2010: 42C05, 33C50

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[1]Departamento de Matemática Aplicada, Universidad de Granada, Granada, Spain
amdelgado@ugr.es, tperez@ugr.es, mpinar@ugr.es

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Nilpotent systems with an inverse integrating factor

Antonio Algaba[1], **Isabel Checa**[1], **Cristóbal García**[1], **Manuel Reyes**[1]

En este trabajo caracterizamos los sistemas nilpotentes cuyo primer término cuasi-homogéneo asociado al tipo $(1, n)$ es $(y, \sigma x^n)^T$, con $\sigma = \pm 1$; los cuales tienen un factor integrante inverso sobre $\mathbb{C}((x, y))$. En estos casos, podemos determinar que estos sistemas admiten un factor integrante inverso de la forma $(h + \ldots)^q$ siendo $h = 2\sigma x^{n+1} - (n + 1)y^2$ y q un número racional.

Demostramos, para n par, que los sistemas con factor integrante inverso formal son formalmente equivalente a $(\dot{x}, \dot{y})^T = (y, x^n)^T$. En el caso de n impar, damos una forma normal formal que los caracteriza. Como consecuencia, damos una relación entre la existencia de factor integrante inverso formal, problema de centro e integrabilidad de los sistemas considerados.

Keywords: punto singular degenerado, factor integrante inverso, formas normales cuasi-homogéneas

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[1]Departamento de Matemáticas
Universidad de Huelva
Facultad de Ciencias Experimentales. Avda. Tres de Marzo s/n, 21071 Huelva, España.
algaba@uhu.es, isabel.checa@dmat.uhu.es,
cristoba@uhu.es, colume@uhu.es

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On Sobolev orthogonality on the unit ball

Teresa E. Pérez[1], **Miguel A. Piñar**[1], **and Yuan Xu**[2]

For the weight function $W_\mu(x) = (1 - \|x\|^2)^\mu$, $\mu > -1$, $\lambda > 0$, and ω_μ a normalizing constant, a family of mutually orthogonal polynomials on the unit ball with respect to the inner product

$$\langle f, g\rangle = \omega_\mu \left[\int_{S^{d-1}} f(\xi)g(\xi)d\sigma(\xi) + \lambda \int_{B^d} \nabla f(x) \cdot \nabla g(x) W_\mu(x)dx \right]$$

are constructed in terms of spherical harmonics and Jacobi orthogonal polynomials of one variable.

Keywords: Sobolev orthogonal polynomials, unit ball, gradient

Mathematics Subject Classification 2010: 33C50, 42C10

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[1]Departamento de Matemática Aplicada
Universidad de Granada
18071. Granada
tperez@ugr.es, mpinar@ugr.es

[2]Department of Mathematics
University of Oregon
Eugene, Oregon 97403-1222
yuan@math.uoregon.edu

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On the roots of the generalized Wills functional

Jesús Yepes Nicolás[1], María A. Hernández Cifre[1]

In 1973 [2] Wills introduced and studied the functional $W(K) = \sum_{i=0}^{n} \mathrm{V}_i(K)$ because of *its relation* with the so called *lattice-point enumerator* $G(K) = \#(K \cap \mathbb{Z}^n)$. Here, $\mathrm{V}_i(K)$ represents the i-th intrinsic volume of K, i.e., the $(n-i)$-th coefficient (up to a constant) of the polynomial (Steiner polynomial) obtained when the volume $\mathrm{vol}(K+\lambda B_n)$ is computed. The Wills functional turns out to have several nice applications, e.g., in Discrete Geometry, where some relations of this functional with the so called *successive minima* of a convex body were given. Hadwiger showed, among others, the following integral representation of $W(K)$:

$$W(K) = \int_{\mathbb{R}^n} e^{-\pi d(x,K)^2} dx, \tag{1}$$

where $d(x, K)$ denotes the Euclidean distance between $x \in \mathbb{R}^n$ and K.

Motivated by this expression, and based on a recent work by Kampf [1], we have generalized this functional for a convex body $K \in \mathcal{K}^n$ with respect to a gauge set $E \in \mathcal{K}^n$, via an (1)-type formula. Moreover, if we change the function $e^{-\pi t^2}$ by another one $G(t)$ properly associated to a measure μ on the nonnegative real line $\mathbb{R}_{\geq 0}$, we may obtain a new polynomial (relative to two convex bodies K and E) which we will call μ-polynomial. For instance, the classical Steiner and Wills polynomials are μ-polynomials for suitable measures μ.

We investigate the roots of these μ-polynomials of convex bodies (and, in particular, the roots of the so called generalized Wills functional). We study its structure, showing that the set of roots in the upper half plane is a closed convex cone, containing the non-positive real axis $\mathbb{R}_{\leq 0}$, and strictly increasing in the dimension (independently on the considered measure μ). Moreover, it is proved that the 'smallest' cone of roots of μ-polynomials is the one given by the Steiner polynomial, which provides, for example, some additional information about the roots of μ-polynomials when the dimension is large enough.

Keywords: Wills functional, Steiner polynomial, intrinsic volumes, roots, successive minima, lattice-point-enumerator, measures

Mathematics Subject Classification 2010: Primary 52A20, 52A39, 52C07, 30C15

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[1]Departamento de Matemáticas
Universidad de Murcia
Campus de Espinardo, 30100 Murcia
jesus.yepes@um.es, mhcifre@um.es

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The core-center of the airport game*

Julio González-Díaz[1], Miguel Ángel Mirás Calvo[2], Carmen Quinteiro Sandomingo[2], Estela Sánchez Rodríguez[3]

The airport problems, see [2], are a well known class of cost allocation problems. The game-theoretic approach consists in transforming these problems into coalitional games, finding a payoff vector that solves the game and studying the allocation corresponding to it. In this work, we choose the core-center, introduced in [1], as our payoff vector. First, explicit integral formulae for the core-center of the airport game are derived. Next, following the survey [3], we analyze the properties of the allocation rule corresponding to the core-center. Special emphasis is put on the monotonicity properties. Finally, we provide a comparison between the Shapley value, the nucleolus, and the core-center for the airport game.

Keywords: Cooperative TU games, Airport games, Core-center

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[1]Departamento de Estatística e Investigación Operativa
Universidade de Santiago de Compostela
julio.gonzalez@usc.es

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[2]Departamento de Matemáticas
Universidade de Vigo
mmiras@uvigo.es, quinteir@uvigo.es

[3]Departamento de Estatística e Investigación Operativa
Universidade de Vigo
esanchez@uvigo.es

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Topological type of Schottky principal G-bundles over compact Riemann surfaces

Susana Ferreira[1], **Carlos Florentino**[2], **Ana Casimiro**[3]

In this poster we will expose some results concerning with the concept of topological type of principal G-bundles and in particular, for the case of Schottky G-bundles over a compact Riemann surface of genus greater or equal to 1 where G is a connected reductive algebraic group over the complex numbers.

Ramanathan introduced, in his paper [4] , the notion of topological type related to a principal G-bundle over a compact Riemann surface. This notion allows us to consider the moduli space of semistable G-bundles as disjoint union of its connected components $\mathcal{M}_G^{ss} = \coprod_{d\in\pi_1(G)} \mathcal{M}_G^{ss,d}$. Since then, a lot of people have studied these matters, specially counting the number of connected components for different types of algebraic groups G (see for example [3]).

In this work we show that in the case of Schottky principal G-bundles where G is a connected reductive algebraic group over $\mathbb{C}$, we can restrict to the case of the connected component of the identity, that is, $\mathcal{M}_G^{ss,\,0}$.

Keywords: Topological type, fundamental group, Principal G-bundles, representation variety, moduli space, reductive algebraic group.

Mathematics Subject Classification 2010: 14D22, 14L24, 14L10

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[1]Escola Superior de Tecnologia e Gestão, Instituto Politécnico de Leiria
susfer@ipleiria.pt

[2]Departmento de Matemática, Instituto Superior Técnico
cfloren@ipleiria.pt

[3]Departmento de Matemática, FCT da Universidade Nova de Lisboa
amc@fct.unl.pt

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Un método de proyección híbrido volúmenes finitos/ elementos finitos para flujos a bajo número de Mach con transporte de especies*

A. Bermúdez[1], **M. Cobas**[1], **J.L. Ferrín**[1], **L. Saavedra**[2] & **M.E. Vázquez-Cendón**[1]

El propósito de este trabajo es presentar un método de proyección híbrido combinando la metodología de volúmenes finitos y elementos finitos para resolver flujos a bajo número Mach (ver [3] y [4])). Partiendo de un mallado del dominio computacional de elementos finitos, se construye la malla dual de volúmenes finitos. Esta malla tiene definidos los nodos en los baricentros de las caras de los tetraedros de elementos finitos (ver [2]), y supone una extensión tridimensional a la presentada en [1]. En una primera etapa se resuelve la ecuación que modela los procesos de transporte y difusión mediante el método de volúmenes finitos. La discretización en tiempo se realiza aplicando el esquema explícito de Euler. El descentramiento (*upwinding*) de los términos convectivos se hace con las resolventes de Riemann clásicas correspondientes a los Q-esquemas de van leer y Rusanov (ver, por ejemplo, [6]). En la segunda etapa de proyección, la corrección de la presión se calcula mediante elementos finitos asociados a la malla inicial de tetraedros. En el presente trabajo se incluyen las ecuaciones de transporte de especies y se analizan los resultados obtenidos empleando una resolución acoplada o desacoplada con el modelo de flujos a bajo número de Mach. Los resultados numéricos se presentan comparados tanto con soluciones analíticas y como con problemas físicos de los que se dispone de resultados experimentales conocidos en la literatura ([5]).

Keywords: Low-Mach number flows; projection method, finite volume method, finite element method;

Mathematics Subject Classification 2010: 76D05, 76M10, 65M08

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[1]Departamento de Matemática Aplicada
Universidad de Santiago de Compostela
Facultad de Matemáticas
1 5782 Santiago de Compostela, Spain
http://www.usc.es/ingmat/
alfredo.bermudez@usc.es
marcos.cobas.garcia@gmail.com
joseluis.ferrin@usc.es
elena.vazquez.cendon@usc.es

[2]Departamento Fundamentos Matemáticos
Universidad Politécnica de Madrid
E.T.S.I. Aeronáuticos. 28040 Madrid, Spain
laura.saavedra@upm.es

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Un resultado de existencia para un modelo de levitación electromagnética en coordenadas cilíndricas

Rafael Muñoz-Sola[1], **Carlos Reales**[2], **Rodolfo Rodríguez**[3]

Presentamos un resultado de existencia de solución para un modelo de la levitación de un cuerpo metálico cilíndrico sometido a un campo electromagnético inducido por una bobina cilíndrica. Usamos el modelo transitorio de corrientes inducidas escrito en coordenadas cilíndricas (r, θ, z). Suponemos que la intensidad $\vec{J}_S$ de la fuente de corriente en la bobina es un dato, que $\vec{J}_S = J_S(t, r, z)\vec{e}_\theta$ y que todas las cantidades físicas son independientes de θ. Introducimos el potencial magnético $\vec{A} = A(t, r, z)\vec{e}_\theta$ y, procediendo como en [1], reducimos este modelo a la EDP parabólica degenerada

$$\sigma \frac{\partial A}{\partial t}\vec{e}_\theta + \nabla \times \left[\frac{1}{\mu}\nabla \times (A\vec{e}_\theta)\right] = J_S\vec{e}_\theta \quad \text{en } (0, T) \times \Omega, \tag{1}$$

donde $\Omega = (0, r_{max}) \times (z_{min}, z_{max})$ está en el semiplano meridiano (r, z), $\mu > 0$ es constante, $\Omega_t = \Omega_0 + u(t)\vec{e_z}$ es la posición ocupada por la pieza metálica en el instante t, σ es la conductividad eléctrica de la pieza metálica en Ω_t y $\sigma = 0$ fuera de Ω_t. Suponemos que $\sigma(t, r, z) = \hat{\sigma}((r, z) - u(t)\vec{e_z})$, donde $\hat{\sigma} \in L^\infty(\Omega_0)$ y $\hat{\sigma} \geq \underline{\sigma} > 0$. Suplementamos la EDP (1) con las condiciones $A = 0$ sobre $(0, T) \times \partial\Omega$ y $A|_{\{0\}\times\Omega_0} = A^0$. El desplazamiento vertical u de la pieza metálica está gobernado por el problema de valor inicial

$$m\frac{d^2u}{dt^2}(t) = 2\pi \int_{\Omega_t} f_z(t, r, z)\, r\, dr\, dz, \qquad \frac{du}{dt}(0) = v_0, \qquad u(0) = 0, \tag{2}$$

donde f_z es la componente vertical de la fuerza de Lorentz, que en este caso está dada por

$$f_z = -\sigma \frac{\partial A}{\partial t}\frac{\partial A}{\partial z} \quad \text{en } \Omega_t. \tag{3}$$

Hemos demostrado en [2] que, bajo hipótesis apropiadas, fijado cualquier $u \in C^1([0, T])$, la EDP (1) tiene una única solución débil, que denotamos $A = A(u)$. Bajo la hipótesis adicional de que el conjunto de discontinuidades de $\hat{\sigma}$ sea de medida cero, demostramos que el problema acoplado tiene al menos una solución local en tiempo. Para ello, aplicamos el teorema de Schauder a la aplicación $F : u \in K \subset C^1([0, T]) \mapsto w \in C^1([0, T])$, donde K es un conjunto convexo cerrado apropiado y w denota la solución del problema de valor inicial anterior con $A = A(u)$.

Keywords: Levitation, transient eddy current problems, axisymmetric problems, degenerate parabolic problems

Mathematics Subject Classification 2010: 35K65

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[1]Departamento de Matemática Aplicada
Universidad de Santiago de Compostela
15782, Santiago de Compostela, España
rafael.munoz@usc.es

[2]Departamento de Matemáticas y Estadística
Universidad de Córdoba
Montería, Colombia
creales@correo.unicordoba.edu.co

[3]CI^{2}MA, Departamento de Ingeniería Matemática,
Universidad de Concepción
Casilla 160-C, Concepción, Chile
rodolfo@ing-mat.udec.cl

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Vector-valued inequalities for fractional integrals for orthonormal systems

Óscar Ciaurri[1]

Let $\{\phi_k^\alpha\}_{k\geq 0}$ be a family of orthonormal systems in the space $L^2(X, d\mu)$. We suppose $\alpha \in [-a, \infty)$, with $a \in \mathbb{R}^+$. If the functions $\{\phi_k^\alpha\}_{k\geq 0}$ are eigenfunctions of a second order differential operator Δ_α, i. e.,

$$\Delta_\alpha \phi_k^\alpha = \lambda_{k,\alpha} \phi_k^\alpha,$$

then, for each $\sigma > 0$, the fractional integrals are defined by

$$I_\alpha^\sigma f(x) = \sum_{k=0}^{\infty} \frac{a_k(f)}{\lambda_{k,\alpha}^\sigma} \phi_k^\alpha(x), \quad a_k(f) = \int_X f(y) \phi_k^\alpha(y)\, d\mu.$$

We present vector-valued inequalities for the fractional integrals of the following type

$$\Big\| \Big(\sum_{j=0}^{\infty} |I_{\alpha+j}^\sigma f_j|^2 \Big)^{1/2} \Big\|_{L^q(X,d\mu)} \leq C \Big\| \Big(\sum_{j=0}^{\infty} |f_j|^2 \Big)^{1/2} \Big\|_{L^p(X,d\mu)},$$

for Laguerre and Jacobi orthonormal systems.

This work has been done in collaboration with L. Roncal and P. R. Stinga and it can be seen in [1] and [2].

Keywords: Vector-valued inequalities, orthonormal systems, fractional integrals

Mathematics Subject Classification 2010: 42C10

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[1]Departamento de Matemáticas y Computación
Universidad de La Rioja
Edif. Juan Luis Vives, c/ Luis de Ulloa s/n, Logroño, 26003
oscar.ciaurri@unirioja.es