PRACTICAL TOOLS TO SOLVE INDEFINITE PROBLEMS

Patrick Ciarlet, ENSTA/POEMS, France

SUMMARY

In electromagnetism, metamaterials can exhibit real-valued, negative electric permittivity and/or magnetic permeability (in a given frequency range). Then, if one solves an interface problem, between a "classical" material and a metamaterial, the permittivity and/or the permeability exhibit a sign-shift at the interface, which yields an indefinite problem. Mathematically speaking, the bilinear/sesquilinear form of the corresponding variational formulation is neither positive nor negative. In this course, we propose some tools to solve this interface problem. For that, we use the theory of T-coercivity, which amounts to finding some "explicit" operator T, such that the modified form (a(.,.) becomes a(.,T.)) is positive, and even coercive. One can then apply the usual results of functional analysis to solve the problem. Finally, one can discretize the problem using standard discretization methods, and analyze the convergence of the discrete solution towards the exact solution.

OUTLINE

§1 Motivation:
We provide a brief introduction on metamaterials and the interface problems for Maxwell's equations, with either vector unknowns (the electromagnetic fields) or scalar unknowns (the potentials).

§2 General framework:
We recall some results on well-posedness, which we reformulate using T-coercivity. Then, we give some "practical" examples on how to achieve T-coercivity.

§3 Optimality of T-coercivity:
We study how to obtain the "best" results in terms of T-coercivity, namely how to prove well-posedness under the most general assumptions on the parameters (permittivity, permeability) defining the scalar interface problem. We also discuss some instances for which this problem is not well-posed.

§4 Approximation results:
We give an overview of some convergence results, using classical tools of numerical analysis, mixed with T-coercivity.

§5 Discretization of the scalar interface problem:
We illustrate the results of §4 on some particular cases, using a conforming discretization of the exact problem.

§6 Discretization of the scalar interface problem - bis:
We study alternate approaches to solve numerically the problem, using either a DG method, or the discretization of modified (by adding dissipation) problems.