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LETTER TO THE EDITOR

Scaling principles and performance in sport

Les Lois d'échelle appliquées aux performances en sports

1. Introduction

Inspired by the idea that world records for running provide data of physiological significance [1], Hammerman has compared performances of men and women in different sports and event types and distances, in order to see if there is any universal parameter that relates them, independent of the particular capabilities and requirements of any sport [2].

In doing this, Hammerman has found a surprisingly stable ratio: in athletics, for nine indoor distances from 60 m to 3000 m, the quotient of the mean women/men record speeds is 0.90 ± 0.01; and for 21 outdoor distances, from 100 m to marathon, it is 0.89 ± 0.02 (for example, the quotient of the 19.19 s of Usain Bolt and 21.34 s of Florence Griffith-Joyner is 0.899).

This is even more astonishing if this procedure is extended to other sports. Hammerman did it also for kayaking, for different distances and number of paddlers: the mean ratio was 0.88 ± 0.01. For swimming, for a total of 30 categories of events covering all the olympic distances and the four styles, the mean ratio was 0.90 ± 0.02. The same happened in track cycling, holding the same even for the hour record (54.50 km for men, 47.98 km for women, yielding a quotient of 0.88). In rowing, the mean ratio for several categories was 0.90 ± 0.01. In speed skating, it was 0.92 ± 0.01.

In summary, after considering 90 categories in all the aforementioned sports, Hammerman arrived to a shockingly stable trend: the ratio was 0.89 with a standard deviation of only 0.02. And what is more, such ratios seem to have been holding for at least 50 years [2].

The same author suggests that such constant could stem from the ratios in the haemoglobin concentration [2] (in men it is in the range 13.6—17-5 grams per decilitre; in women it is 12.0—15.5, yielding ratios of 0.88 at the low end and 0.89 at the high end [3]).

2. Results and discussion

Here another view is introduced, using basic scaling principles of physical performance that were first hinted at by Galileo [4] and that would apply, in particular, to the running performance in athletics.

A runner's maximum speed, v, depends on the force F applied by the muscles used for running; on the mass m of these muscles that undergo repeated acceleration and deceleration in performing the action of running; and on the displacement d of the point at which the force F is applied. A simple dimensional analysis using Buckingham's theorem [5] shows that the only dimensionless combination of v, F, d and m is $mv^2/(Fd)$, i.e.,

$$v^2 \alpha Fd/m$$

(if we name the spatial dimension as [L], the time as [T] and the mass as [M], then v is [L]/[T] and F is [M][L]/$[T]^2$; therefore v^2 is $([L]/[T])^2$, the same as Fd/m, which is $[M][L][L]/[T]^2[M] = ([L]/[T])^2$).

In order to make a comparative analysis, if we assume an ideal good runner that does not waste effort by bobbing up and down, i.e. by vertical displacement of the centre of mass, his or her total mass is not in itself a key determinant of v [6]. Therefore, v would be determined by the square root of Fd. At this point it is crucial to note that the mean stature ratio female/male across the main developed countries seems to follow another universal human constant: around 0.92 [7].

Now, if we assume that d is proportional to the stature, and if F in humans were proportional to the haemoglobin concentration, then by using the haemoglobin ratio female/male, we would arrive (for running) to the ratio described in [2].

Disclosure of interest

The author declares that he has no competing interest.

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