Tema 4

Desintegraciones y reacciones en el medio estelar

Asignatura de Astrofísica Nuclear

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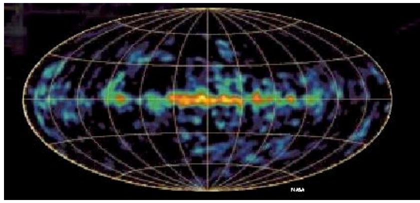
Decays in a stellar plasma

1.1 Gamma-ray transitions

In a hot plasma, excited states in a given nucleus are thermally populated through photon absorption, Coulomb excitations by surrounding ions, inelastic particle scattering or other mechanisms. The time scale for excitation and de-excitation is much shorter than stellar hydrodynamics time scales. Contrary to lab investigations where decaying or reacting nuclei are in their ground state, these excited states will play an important role in stellar decays or reactions. At thermal equilibrium the probability for populating a given state μ can be obtained as:

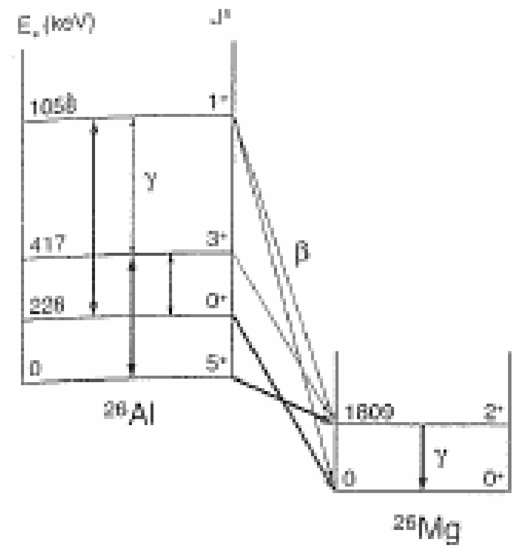
$$P_\mu = \frac{N_\mu}{N} = \frac{g_\mu e^{-E_\mu/kT}}{\sum_\mu g_\mu e^{-E_\mu/kT}}$$

$g=2j+1$ being j the spin of the state μ



The decay of ^{26}Al represents a clear example of the role of excited states in the nuclear media. This nucleus is though to be produced In type II supernovae during the explosive carbon and neon burning phases. This nucleus decays to the first excited state (1809 keV) in ^{26}Mg. The observation of this gamma ray in several γ-ray telescopes as COMPTEL aboard the Gamma Ray Observatory is a major prove for nucleosynthesis processes in the Universe.

Since the first excited state in ^{26}Al is an isomer decaying to the ground state in ^{26}Mg, the observed intensities of 1809 keV γ-rays can only be transformed in nucleosynthesis rate of ^{26}Al if one takes into consideration the populations of the different excited states in ^{26}Al, and in particular the isomeric state.

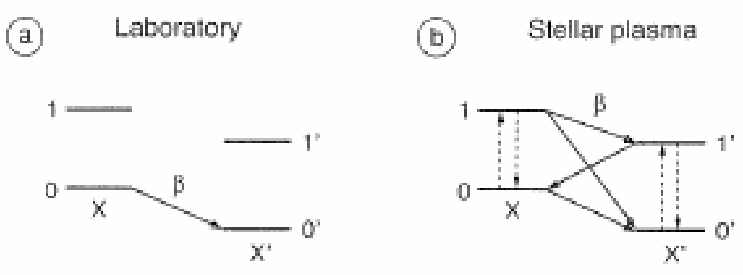


Decays in a stellar plasma

1.2 Weak interactions

β-decay:

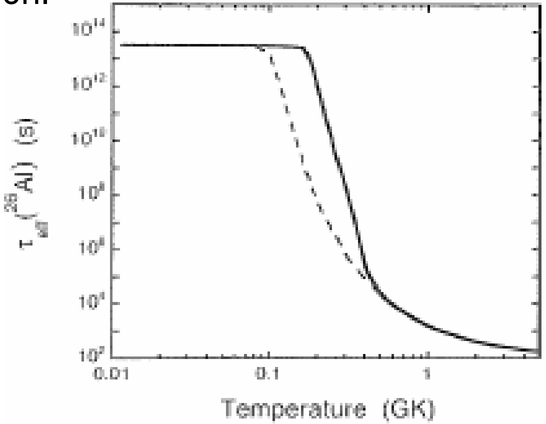
In a hot plasma, excited states in a parent nucleus are thermally populated and may also undergo β-decay transitions to the daughter nucleus. Even stable nuclei may undergo a b-decay in the stellar medium. The total β-decay rate will be given by the weighted sum of the individual transition rates λ_{ij} according to:



$$\lambda^*_\beta = \sum_i P_i \sum_j \lambda_{ij}$$

The sum over *I* and *j* runs over the parent and daughter states and Pi can be obtained from the previous equation.

Under these conditions, β-decay becomes temperature dependent by also density dependent at sufficiently large values of density when the electron gas is degenerate, limiting the number of final states available for the electron emission.



Temperature dependence of the decay rate of ^{26}Al

Decays in a stellar plasma

1.2 Weak interactions

Electron capture:

At the temperature typical of the stellar interior most nuclei posses few, if any, bound electrons. Being their decay constant for bound electron capture very small. However, and due to the density of free electrons in the stellar medium, nuclei can decay through the capture of free electrons. The probability for this process is proportional to the electron density and inversely proportional to the average electron velocity.

At low densities, the kinetic energies of the free electrons are usually small. At very high densities, however, the (Fermi) energy of the degenerate electrons may become sufficiently large to cause nuclei to undergo continuum capture of energetic electrons, even if they are stable in the laboratory.

Pair production:

At high temperatures pair production can become and effective process. Then positron capture should be considered in addition to the continuum electron capture.

Neutrino energy loss mechanism:

This mechanism known as *Urca process* becomes important at high temperatures and densities and consists of alternate electron captures and β^--decays involving the same pair of parent and daughter nuclei being the net result of two subsequent decays of a neutrino anti-neutrino pair.

$$ {}^{A}_{Z}X_{N}(e^{-},\nu)\,{}^{A}_{Z-1}X_{N+1}(\beta^{-},\bar{\nu})\,{}^{A}_{Z}X_{N}\ldots \qquad {}^{A}_{Z}X_{N}+e^{-}\rightarrow {}^{A}_{Z}X_{N}+e^{-}+\nu+\bar{\nu} $$

Cross sections and reaction rates

2.1 Particle induced reactions

In the stellar medium reactions are produced by collisions of two moving particles therefore the reaction rate (reactions per time and volume unit) will be determined by the number of colliding particles per volume unit, their relative velocity and their interaction cross section.

$$r_{01} = N_0 N_1 v \sigma(v)$$

In a stellar plasma at thermodynamic equilibrium the velocity of the constituent particles follows a given distribution. Then we can generalize the expression for the reaction rate taking into account the possibility of having identical colliding particles as:

$$r_{01} = N_0 N_1 \int_0^\infty v P(v) \sigma(v) dv = N_0 N_1 \langle \sigma v \rangle_{01} \equiv \frac{N_0 N_1 \langle \sigma v \rangle_{01}}{(1+\delta_{01})}$$

Since nuclei in the plasma move non relativistically their relative velocities can be described by a Maxwell-Boltzmann distribution, then:

$$P(v)dv = \left(\frac{m_{01}}{2\pi kT}\right)^{3/2} e^{-m_{01} v^2/(2kT)} 4\pi v^2 dv \quad \Rightarrow \quad P(E)dE = \left(\frac{m_{01}}{2\pi kT}\right)^{3/2} e^{-E/kT} 4\pi \frac{2E}{m_{01}} \frac{dE}{m_{01}} \sqrt{\frac{m_{01}}{2E}}$$

The reaction rate per mol is defined as::

$$N_A \langle \sigma v \rangle_{01} = \left(\frac{8}{\pi m_{01}}\right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty E \sigma(E) e^{-E/kT} dE = \frac{3.7318 \cdot 10^{10}}{T_9^{3/2}} \sqrt{\frac{M_0 + M_1}{M_0 M_1}} \int_0^\infty E \sigma(E) e^{-11.605/T_9} dE \quad (cm^3 mol^{-1} s^{-1})$$

With E given in *MeV*, $T_9 = T/10^9$ in kelvin, , M in units of *u* and the cross section in *barn*.

2.2 Photon induced reactions

If one of the colliding particles is a photon the reaction is called photodisintegration (γ+3→0+1). Considering that Photons move with the speed of light we can write the reaction rate and the corresponding decay constant as:

$$r_{\gamma 3} = N_3 \int_0^\infty c N_\gamma(E_\gamma)\sigma(E_\gamma)dE_\gamma \qquad \lambda_\gamma(3) = \frac{r_{\gamma 3}}{N_3} = \int_0^\infty c N_\gamma(E_\gamma)\sigma(E_\gamma)dE_\gamma$$

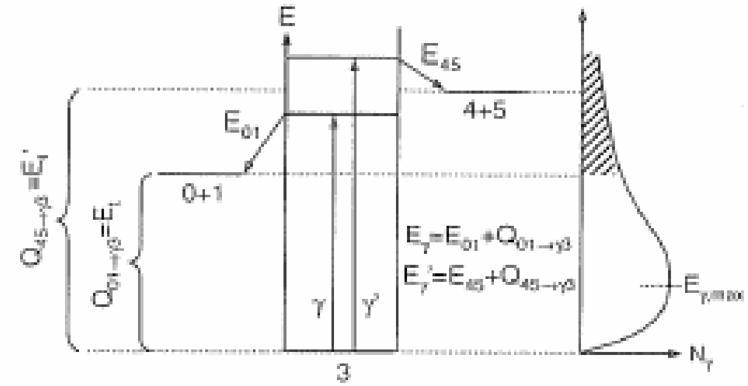
The energy distribution of photons can be obtained from the Planck's radiation law *u(E).*

$$N_\gamma(E_\gamma)dE_\gamma = \frac{u(E_\gamma)}{E_\gamma}dE_\gamma = \frac{8\pi}{(hc)^3}\frac{E_\gamma^2}{e^{E_\gamma/kT}-1}dE_\gamma$$

Then, the final expression for the decay rate will be:

$$\lambda_\gamma(3) = \frac{8\pi}{h^3c^2}\int_0^\infty \frac{E_\gamma^2}{e^{E_\gamma/kT}-1}\sigma(E_\gamma)dE_\gamma$$

In general photodisintegration are endotermic reaction, then the lower integration limit in the previous equation will be $Q_{\gamma 3}$.



2.3 Abundance evolution

The rate of change of the abundance of nucleus 0 due to reactions with nucleus 1 can be expressed as:

$$\left(\frac{dN_0}{dt}\right)_1 = -\lambda_1(0)N_0 = -\frac{N_0}{\tau_1(0)} = -(1+\delta_{01})r_{01} = -N_0 N_1 \langle\sigma v\rangle_{01}$$

From these equations we obtain the following relations:

$$r_{01} = \frac{\lambda_1(0)N_0}{(1+\delta_{01})} = \frac{1}{(1+\delta_{01})}\frac{N_0}{\tau_1(0)}$$

$$\tau_1(0) = \frac{N_0}{(1+\delta_{01})r_{01}} = \frac{1}{N_1\langle\sigma v\rangle_{01}} = \left(\rho\frac{X_1}{M_1}N_A\langle\sigma v\rangle_{01}\right)^{-1}$$

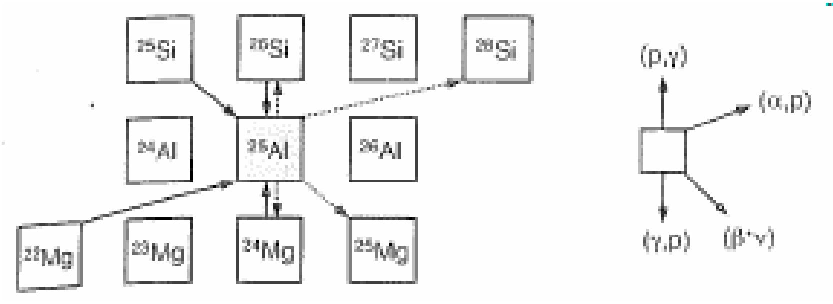
$$\lambda_1(0) = \frac{1}{\tau_1(0)} = N_1\langle\sigma v\rangle_{01} = \rho\frac{X_1}{M_1}N_A\langle\sigma v\rangle_{01}$$

The decay constant of a nucleus for destruction via particle-induced reactions depends explicitly on the stellar density and implicitly on temperature. If a nucleus 0 can be destroyed by different reactions its total lifetime is:

$$\frac{1}{\tau(0)} = \sum_i \frac{1}{\tau_i(0)}$$

Cross sections and reaction rates

2.3 Abundance evolution



The final abundance of a nucleus can be obtained taking into account all creation and destruction mechanisms (reactions, β-decays, photodisintegrations,…)

$$\frac{dN_i}{dt} = \left[\sum_{j,k} N_j N_k \langle \sigma v \rangle_{jk \to i} + \sum_{l} \lambda_{\beta, l \to i} N_l + \sum_{m} \lambda_{\gamma, m \to i} N_m \right]$$

$$- \left[\sum_{n} N_n N_i \langle \sigma v \rangle_{ni} + \sum_{o} \lambda_{\beta, i \to o} N_i + \sum_{p} \lambda_{\gamma, i \to p} N_i \right]$$

In a realistic situation we should consider the evolution of not just one nuclide, but of several species simultaneously. Such a system of coupled, nonlinear ordinary differential equations is called a *nuclear reaction network.* Very often, equilibrium conditions together with the reciprocity theorem helps in solving these nuclear reaction networks.

2.4 Forward and reverse reactions

The cross sections of a forward and reverse reaction are related by the reciprocity theorem. If we consider Reactions involving particles with rest mass 0+1→2+3 we obtain the following relation:

$$\frac{\sigma_{23\to01}}{\sigma_{01\to23}} = \frac{(2j_0+1)(2j_1+1)}{(2j_2+1)(2j_3+1)}\frac{m_{01}E_{01}}{m_{23}E_{23}}\frac{(1+\delta_{23})}{(1+\delta_{01})}$$

$$\frac{N_A\langle\sigma v\rangle_{23\to01}}{N_A\langle\sigma v\rangle_{01\to23}} = \left(\frac{m_{01}}{m_{23}}\right)^{1/2}\frac{\int_0^\infty E_{23}\sigma_{23\to01}e^{-E_{23}/kT}dE_{23}}{\int_0^\infty E_{01}\sigma_{01\to23}e^{-E_{01}/kT}dE_{01}} = \frac{(2j_0+1)(2j_1+1)(1+\delta_{23})}{(2j_2+1)(2j_3+1)(1+\delta_{01})}\left(\frac{m_{01}}{m_{23}}\right)^{2/3} e^{-Q_{01\to23}/kT}$$

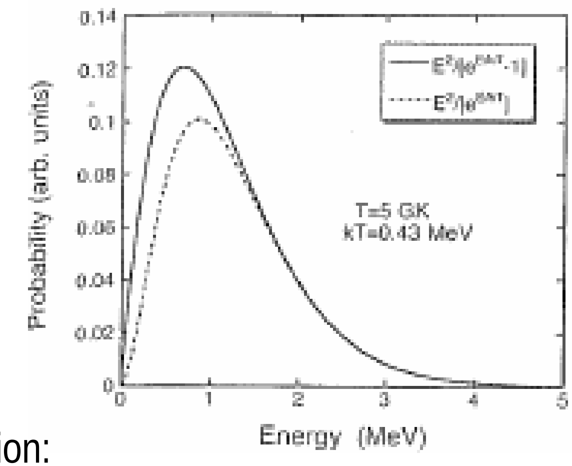
For reactions involving photons, 0+1→γ+3, we obtain these other expressions:

$$\frac{\sigma_{\gamma3\to01}}{\sigma_{01\to\gamma3}} = \frac{(2j_0+1)(2j_1+1)}{2(2j_3+1)}\frac{m_{01}c^2E_{01}}{E_\gamma^2}\frac{1}{(1+\delta_{01})}$$

$$\frac{\lambda_\gamma(3)}{N_A\langle\sigma v\rangle_{01\to\gamma3}} = \frac{\frac{8\pi}{h^3c^2}\int_0^\infty \frac{E_\gamma^2}{e^{E_\gamma/kT}-1}\frac{(2j_0+1)(2j_1+1)}{(2j_3+1)(1+\delta_{01})}\frac{m_{01}c^2E_{01}}{E_\gamma^2}\sigma_{01\to\gamma3}dE_\gamma}{\left(\frac{8}{\pi m_{01}}\right)^{1/2}\frac{N_A}{(kT)^{3/2}}\int_0^\infty E_{01}\sigma_{01\to\gamma3}e^{-E_{01}/kT}dE_{01}}$$

Cross sections and reaction rates

2.4 Forward and reverse reactions

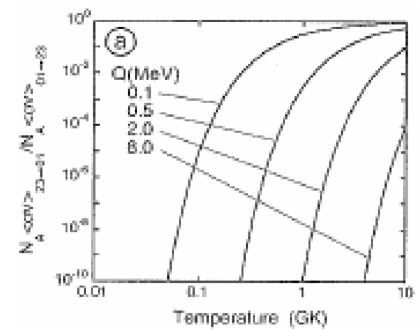


Since most of the capture reactions are not only exothermic, but their Q values are relatively large, then the integration over the gamma Energy will have as lower limit $Q_{01\rightarrow\gamma 3}$, this implies E$\gamma$>>kT and we may use the following approximation:

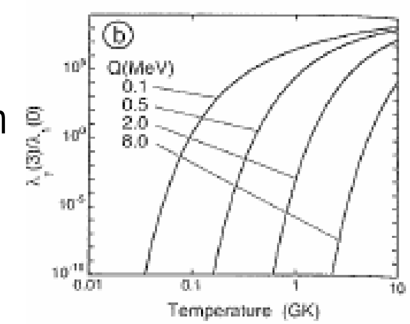
$$e^{E_\gamma/kT}-1\approx e^{E_\gamma/kT}$$

Then we can find an analytical relation relating capture and photodisintegration:

$$\frac{\lambda_\gamma(3)}{N_A\langle\sigma v\rangle_{01\rightarrow\gamma 3}}=\left(\frac{2\pi}{h^2}\right)^{3/2}\frac{(m_{01}kT)^{3/2}}{N_A}\frac{(2j_0+1)(2j_1+1)}{(2j_3+1)(1+\delta_{01})}e^{-Q_{01\rightarrow\gamma 3}/kT}$$



For reactions involving particles and Q>0, the reverse reaction becomes important at sufficiently large temperatures and at small Q-values.



In the case of reactions involving gammas, the capture process dominates in reactions leading to even-even nuclei (larger Q-values). Then, the net effect of photodisintegration In stellar plasmas at elevated temperatures is to convert nuclei to more stable species. This result will be specially important for the advanced burning stages in massive stars.

Cross sections and reaction rates

2.5 Reaction rates at elevated temperature

At elevated temperatures, reacting nuclei will be thermally excited. For a given reaction 0+1→2+3 the rate Including thermally excited states is obtained by summing over all transitions to relevant excited states in nuclei 2 and 3, and averaging over excited states in nuclei 0 and 1. Considering 1 and 2 as light particles and neglecting their excited states one obtains:

$$N_A\langle\sigma v\rangle^{*}_{01\to 23} = \sum_{\mu} P_{0\mu} \sum_{\nu} N_A\langle\sigma v\rangle^{\mu\to\nu}_{01\to 23} = \frac{\sum_{\mu} g_0 e^{-E_o/kT} \sum_{\nu} N_A\langle\sigma v\rangle^{\mu\to\nu}_{01\to 23}}{\sum_{\mu} g_{o\mu} e^{-E_{o\mu}/kT}}$$

Where μ and ν labels for states in the target (0) and residual (3) nuclei and: $N_A\langle\sigma v\rangle_{01\to 23} = \sum_{\nu} N_A\langle\sigma v\rangle^{g.s.\to\nu}_{01\to 23}$

The ralation between the reaction rates including excited states and those measured at the laboratory is:

$$N_A\langle\sigma v\rangle^{*}_{01\to 23} = R_{tt} N_A\langle\sigma v\rangle_{01\to 23} = \frac{\sum_{\mu} g_{0\mu} e^{-E_{0\mu}/kT} \dfrac{\sum_{\nu} N_A\langle\sigma v\rangle^{\mu\to\nu}_{01\to 23}}{\sum_{\nu} N_A\langle\sigma v\rangle^{g.s\to\nu}_{01\to 23}}}{\sum_{\mu} g_0 e^{-E_{0\mu}/kT}} N_A\langle\sigma v\rangle_{01\to 23}$$

The relation between the forward and reverse stellar rate will be:

$$\frac{N_A\langle\sigma v\rangle^{\nu\to\mu}_{23\to 01}}{N_A\langle\sigma v\rangle^{\mu\to\nu}_{01\to 23}} = \frac{g_{0\mu} g_1 (1+\delta_{23})}{g_{3\nu} g_2 (1+\delta_{01})} \left(\frac{m_{01}}{m_{23}}\right)^{3/2} e^{-Q^{\mu\to\nu}_{01\to 23}/kT}$$

Cross sections and reaction rates

2.5 Reaction rates at elevated temperature

Summing over all final states we obtain:

$$\frac{N_A\langle\sigma v\rangle^*_{23\to01}}{N_A\langle\sigma v\rangle^*_{01\to23}} = \left(\frac{m_{01}}{m_{23}}\right)^{3/2} \frac{g_0 g_1 G_0^{norm}}{g_2 g_3 G_3^{norm}} e^{-Q_{01\to23}/kT}$$

$$G_i^{norm} = \frac{G_i}{g_i} = \frac{\sum_\mu g_{i\mu} e^{-E_{i\mu}/kT}}{g_i}$$

where we have introduced the *normalized partition function* G_i with $g_{i\mu}$ and $E_{i\mu}$ the statistical weigth and excitation energy of the state μ in nucleus *i*, and g_i the statistical weigth of the ground state of nucleus *i*.

We can easily generalize this result by allowing excitations in nuclei 1 and 2 for reactions involving only particles with rest mass as:

$$\frac{N_A\langle\sigma v\rangle^*_{23\to01}}{N_A\langle\sigma v\rangle^*_{01\to23}} = \frac{(2j_0+1)(2j_1+1)(1+\delta_{23})}{(2j_2+1)(2j_3+1)(1+\delta_{01})}\left(\frac{G_0^{norm}G_1^{norm}}{G_2^{norm}G_3^{norm}}\right)\left(\frac{M_0 M_1}{M_2 M_3}\right)^{3/2} e^{-11.605 Q_{01\to23}/T_9}$$

and for reactions involving photons:

$$\frac{\lambda_\gamma(3\to01)}{N_A\langle\sigma v\rangle^*_{01\to\gamma3}} = 9.8685\cdot10^9\, T_9^{3/2}\frac{(2j_0+1)(2j_1+1)}{(1j_3+1)(1+\delta_{01})}\left(\frac{G_0^{norm}G_1^{norm}}{G_3^{norm}}\right)\left(\frac{M_0 M_1}{M_3}\right)^{3/2} e^{-11.605 Q_{01\to23}/T_9}$$

Cross sections and reaction rates

2.6 Reaction rate equilibria

The net reaction rate considering forward and reverse reactions is obtained as:

$$r = r_{01\to 23} - r_{23\to 01} = \frac{N_0 N_1 \langle \sigma v \rangle_{01\to 23}}{(1+\delta_{01})} - \frac{N_2 N_3 \langle \sigma v \rangle_{23\to 01}}{(1+\delta_{23})}$$

being the equilibrium condition (*r=0*):

$$\frac{N_2 N_3}{N_0 N_1} = \frac{(1+\delta_{23})\langle \sigma v \rangle_{01\to 23}}{(1+\delta_{01})\langle \sigma v \rangle_{23\to 01}} = \frac{(2j_2+1)(2j_3+1)}{(2j_0+1)(2j_1+1)} \frac{G_2^{norm} G_3^{norm}}{G_0^{norm} G_1^{norm}} \left(\frac{m_{23}}{m_{01}}\right)^{2/3} e^{Q_{01\to 23}/kT}$$

while for reactions involving photons:

$$r = r_{01\to \gamma 3} - r_{\gamma 3\to 01} = \frac{N_0 N_1 \langle \sigma v \rangle_{01\to \gamma 3}}{(1+\delta_{01})} - \lambda_\gamma(3) N_3$$

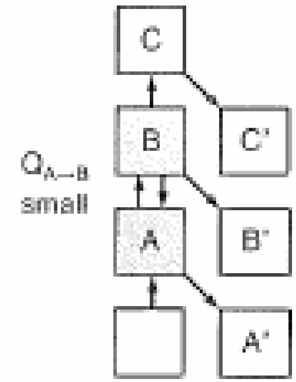
$$\frac{N_3}{N_0 N_1} = \frac{1}{(1+\delta_{01})} \frac{\langle \sigma v \rangle_{01\to \gamma 3}}{\lambda_\gamma(3)} = \left(\frac{h^2}{2\pi}\right)^{2/3} \frac{1}{(m_{01} kT)^{3/2}} \frac{(2j_3+1)}{(2j_0+1)(2j_1+1)} \frac{G_3^{norm}}{G_0^{norm} G_1^{norm}} e^{Q_{01\to \gamma 3}/kT}$$

This last expression is known as the *Saha statistical equation*.

Cross sections and reaction rates

2.6 Reaction rate equilibria

The equilibrium condition can be generalized to several nuclei produced by subsequent reactions or β decays. Assuming that nuclei A and B are in equilibrium and photodisintegration of C is negligible, the transformation of A into C by double capture or B' by capture plus β decay



$$\lambda_{A\to B\to(C\ or\ B')} = \frac{N_B^e}{N_A^e}(\lambda_{B\to C}+\lambda_{B\to B'})$$

N_A and N_B denote the equilibrium abundances of A and B

using the Saha equations:

$$\frac{N_B}{N_A N_a} = \frac{\langle\sigma v\rangle_{A\to B}}{\lambda_{B\to A}} = \frac{1}{N_a}\frac{\lambda_{A\to B}}{\lambda_{B\to A}} = \left(\frac{h^2}{2\pi}\right)^{3/2}\frac{1}{(m_{Aa}kT)^{3/2}}\frac{(2j_B+1)}{(2j_{A+1})(2j_a+1)}\frac{G_B^{norm}}{G_A^{norm}G_a^{norm}}e^{Q_{A\to B}/kT}$$

Thus:

$$\lambda_{A\to B\to(C\ \mathrm{or}\ B')} = \frac{\lambda_{A\to B}}{\lambda_{B\to A}}(\lambda_{B\to A}+\lambda_{B\to B'}) = N_a\left(\frac{h^2}{2\pi}\right)^{3/2}\frac{1}{(m_{Aa}kT)^{3/2}}\frac{(2j_B+1)}{(2j_A+1)(2j_a+1)}\frac{G_B^{norm}}{G_A^{norm}G_a^{norm}}e^{Q_{A\to B}/kT}(\lambda_{B\to C}+\lambda_{B\to B'})$$

Numerically we find:

$$\lambda_{A\to B\to(C\ \mathrm{or}\ B')} = 1.0133\cdot10^{-10}\rho\frac{X_a}{M_a}\left(\frac{M_B}{M_A M_a}\right)^{3/2}\frac{g_B}{g_A g_a}\frac{G_B^{norm}}{G_A^{norm}G_a^{norm}}T_9^{-3/2}e^{11.605Q_{A\to B}/T_9}(\lambda_{B\to C}+\lambda_{B\to B'})$$

2.7 Nuclear energy generation

The nuclear energy generation in stars is given by the Q value of the thermonuclear reactions taking place in the stellar medium:

$$\varepsilon_{01\to 23} = \frac{Q_{01\to 23} r_{01\to 23}}{\rho} = \frac{Q_{01\to 23}}{\rho} \frac{N_0 N_1 \langle \sigma v \rangle_{01\to 23}}{(1+\delta_{01})} = -\frac{Q_{01\to 23}}{\rho(1+\delta_{01})} \left(\frac{dN_0}{dt} \right)_1$$

At higher temperatures also the reverse process must be considered being the net energy production:

$$\varepsilon = \varepsilon_{01\to 23} - \varepsilon_{23\to 01}$$

The time integrated released energy if obtained from:

$$\int \varepsilon_{01\to 23} dt = -\int_{N_0^{init}}^{N_0^{final}} \frac{Q_{01\to 23}}{\rho(1+\delta_{01})} (dN_0)_1 = \frac{Q_{01\to 23}}{\rho(1+\delta_{01})} (\Delta N_0)_1$$

$$(\Delta N_0)_1 = N_0^{initial} - N_0^{final}$$

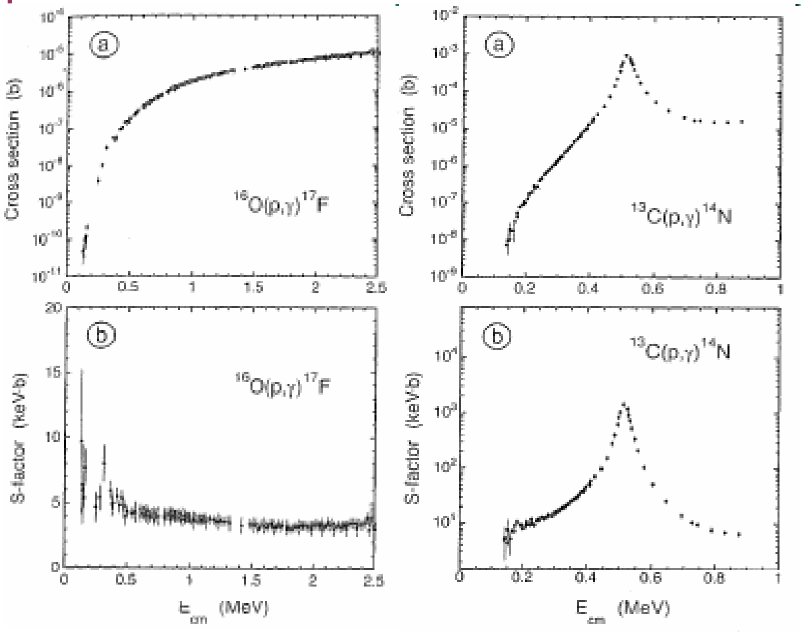
Nonresonant thermonuclear reactions

3.1 Nonresonant reactions induced by charged particles

Absorption cross section of charged particles are dominated by the *1/k²=1/E* dependence term and the transmission coefficient of the Coulomb barrier T_l

$$\sigma_{abs} = \frac{\pi}{k^2}\sum_{l=0}^{l_{max}}(2l+1)T_l$$

$$T_{l=0} \approx \exp\left(-\frac{2\pi}{\hbar}\sqrt{\frac{m}{2E}}Z_1Z_2e^2\right) \equiv e^{-2\pi\eta} \qquad \eta = 0.989534 Z_1Z_2\sqrt{\frac{1}{E}\frac{M_1M_2}{M_1+M_2}}$$



Astrophysical S-factor

In order to performed reliable extrapolations of the measured cross sections at energies of astrophysical interest nuclear Astrophysics introduces the astrophysics S-factor removing the 1/E dependence of the absorption cross sections And the s-wave Coulomb barrier probability.

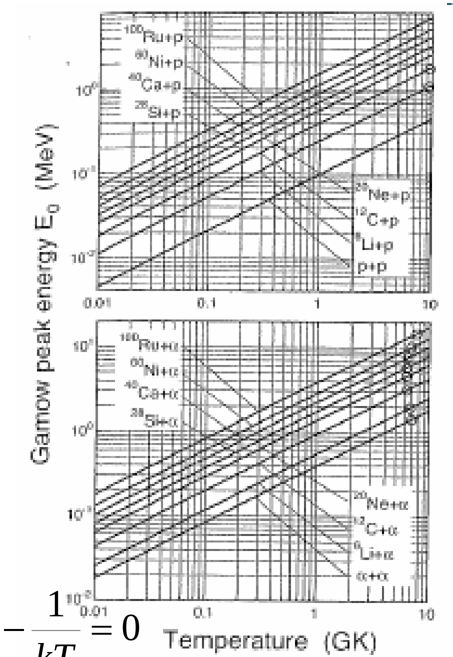
$$\sigma(E) = \frac{1}{E}e^{-2\pi\eta}S(E)$$

Nonresonant thermonuclear reactions

3.1 Nonresonant reactions induced by charged particles

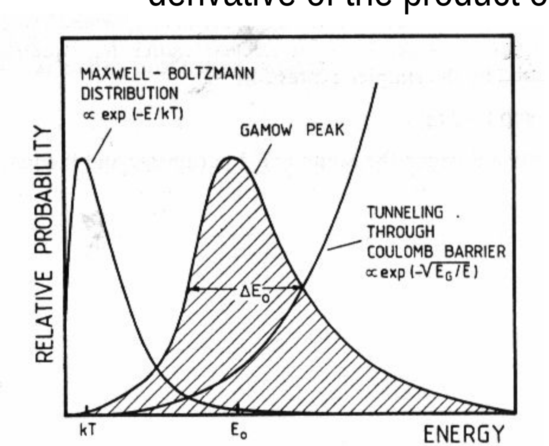
With the definition of the *S*-factor we can write the nonresonant reaction rate as:

$$N_A\langle\sigma v\rangle = \left(\frac{8}{\pi m_{01}}\right)^{1/2} \frac{N_A}{(kT)^{3/2}} \int_0^\infty e^{-2\pi\eta} S(E) e^{-E/kT} dE \approx \left(\frac{8}{\pi m_{01}}\right)^{1/2} \frac{N_A}{(kT)^{3/2}} S_0 \int_0^\infty e^{-2\pi\eta} e^{-E/kT} dE$$



The Gamow peak:

The major contribution to the reaction rate will come from energies where the product of the *Maxwell-Boltzmann distribution* and the *Gamow factor* is near its maximum. This product is known as the *Gamow peak* and represents the relatively narrow energy range over which most nuclear reactions occur in a stellar plasma. The location of the maximum of the Gamow peak ($E=E_0$) can be obtained from the derivative of the product of these two terms:



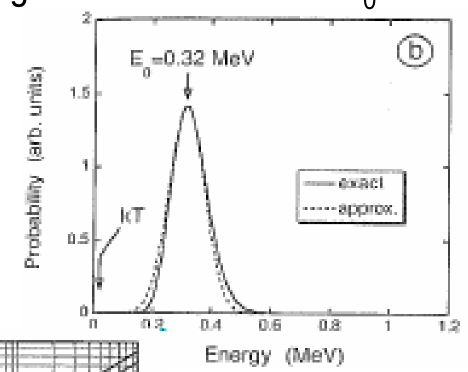
$$\frac{d}{dE}\left(-\frac{2\pi}{\mathrm{h}}\sqrt{\frac{m_{01}}{2E}} Z_0 Z_1 e^2 - \frac{E}{kT}\right)_{E=E_0} = \frac{\pi}{\mathrm{h}} Z_0 Z_1 e^2 \sqrt{\frac{m_{01}}{2}} \frac{1}{E_0^{3/2}} - \frac{1}{kT} = 0$$

$$E_0 = \left[\left(\frac{\pi}{\mathrm{h}}\right)^2 \left(Z_0 Z_1 e^2\right)^2 \left(\frac{m_{01}}{2}\right) (kT)^2\right]^{1/3} = 0.122\left(Z_0^2 Z_1^2 \frac{M_0 M_1}{M_0 + M_1} T_9^2\right)^{1/3}$$

Nonresonant thermonuclear reactions

3.1 Nonresonant reactions induced by charged particles

The shape of the Gamow peak can be approximated by a Gaussian distribution having the maximum at $E=E_0$

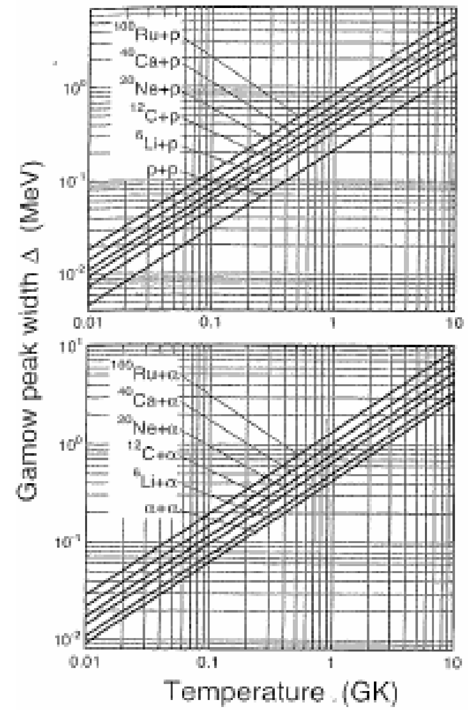


$$\exp\left(-\frac{2\pi}{\mathrm{h}}\sqrt{\frac{m_{01}}{2E}}Z_0Z_1e^2-\frac{E}{kT}\right)=\exp\left(-\frac{2E_0^{3/2}}{\sqrt{EkT}}-\frac{E}{kT}\right)\approx\exp\left(-\frac{3E_0}{kT}\right)\exp\left[-\left(\frac{E-E_0}{\Delta/2}\right)^2\right]$$

Where the *1/e* width Δ of the Gaussian is obtained from the requirement that The second derivatives match at E_0.

$$\frac{d^2}{dE^2}\left(\frac{2E_0^{3/2}}{\sqrt{EkT}}+\frac{E}{kT}\right)_{E=E_0}=\frac{3}{2}\frac{1}{E_0kT}$$

$$\frac{d^2}{dE^2}\left(\frac{E-E_0}{\Delta/2}\right)^2_{E=E_0}=\frac{2}{(\Delta/2)^2}$$



Setting the last two expressions equal and solving for Δ gives:

$$\Delta=\frac{4}{\sqrt{3}}\sqrt{E_0kT}=0.2368\left(Z_0^2Z_1^2\frac{M_0M_1}{M_0+M_1}T_9^5\right)^{1/6}\qquad(\mathrm{MeV})$$

Nonresonant thermonuclear reactions

3.1 Nonresonant reactions induced by charged particles

The nonresonant thermonuclear reaction rates can be calculated by replacing the Gamow peak with a Gaussian:

$$N_A\langle\sigma v\rangle = \left(\frac{8}{\pi m_{01}}\right)^{1/2} \frac{N_A}{(kT)^{3/2}} S_0 \int_0^\infty e^{-2\pi\eta} e^{-E/kT} dE \approx \left(\frac{8}{\pi m_{01}}\right)^{1/2} \frac{N_A}{(kT)^{3/2}} S_0 e^{-3E_0/kT} \int_0^\infty \exp\left[-\left(\frac{E-E_0}{\Delta/2}\right)\right] dE$$

The lower integration limit can be extended to minus infinity without introducing a significant error. In that case The value of the integral will be $\pi^{1/2}\Delta/2$ then:

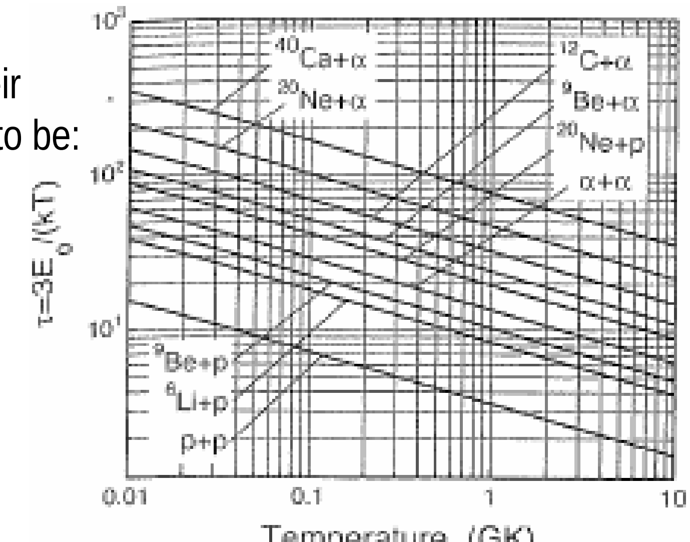
$$N_A\langle\sigma v\rangle = N_A\sqrt{\frac{2}{m_{01}}} \frac{\Delta}{(kT)^{3/2}} S_0 e^{-3E_0/kT}$$

One of the most striking features of thermonuclear reaction rates is their temperature dependence that near some energy $T=T_0$ can be derived to be:

$$N_A\langle\sigma v\rangle_T = N_A\langle\sigma v\rangle_{T_0} (T/T_0)^{(\tau-2)/3}$$

where:

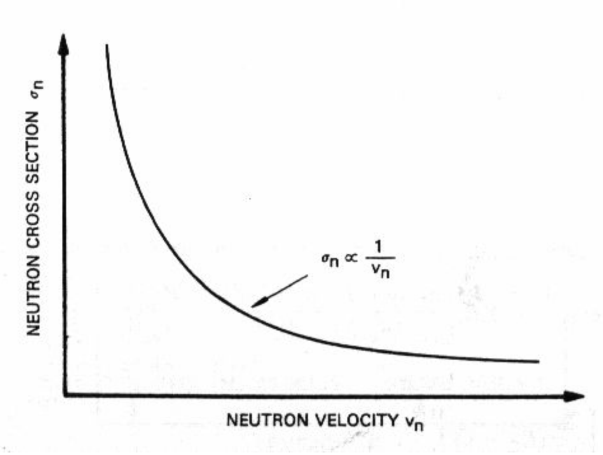
$$\tau = \frac{3E_0}{kT} = 4.2487\left(Z_0^2 Z_1^2 \frac{M_0 M_1}{M_0+M_1} \frac{1}{T_9}\right)^{1/3}$$



Nonresonant thermonuclear reactions

3.2 Nonresonant reactions induced by neutrons

Neutrons that are produced in a star quickly thermalize and their velocities are given by Maxwell-Boltzmann distributions. Altough neutron induced reactions can lead to the emission of charged particles (e.g. n,p) in general these are exotermic reaction being the corresponding barrier transmission coefficients contants, then for s waves:

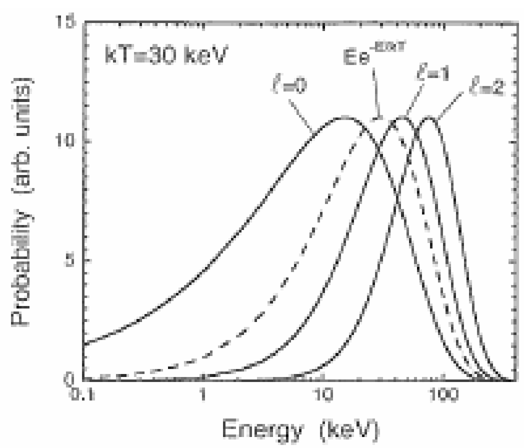


$$\sigma_{abs} = \frac{\pi}{k^2}\sum_{l=0}^{l_{max}}(2l+1)T_l \approx \frac{1}{v} = \frac{1}{\sqrt{E}}$$

Considering higher order partial waves the general expression for the reaction rates of reactions induced by neutrons assuming low neutron energies compared to the neutron binding energy is:

$$N_A\langle\sigma v\rangle = \left(\frac{8}{\pi m_{01}}\right)^{1/2}\frac{N_A}{(kT)^{3/2}}\int_0^\infty E\sigma(E)e^{-E/kT}dE \approx \int_0^\infty E^{l+1/2}e^{-E/kT}dE$$

The integrand $E^{l+1/2}e^{-E/kT}$ represents the stellar energy window in which most of the nonresonant neutron-induced reactions take place. The maximun of the Integrand occurs at $E_{max}=(l+1/2)kT$



Nonresonant thermonuclear reactions

3.2 Nonresonant reactions induced by photons

For nonresonant charged-particle emission reactions induced by photons the decay constant in terms of the astrophysical S-factor is given by the expression:

$$\lambda_\gamma(3) \approx \int_0^\infty \left(E_\gamma - Q_{01\to\gamma 3}\right) e^{-E_\gamma/kT} \frac{e^{-2\pi\eta}}{E_{01}} S(E_{01})\, dE_\gamma \approx S(E_0)\, e^{-Q_{01\to\gamma 3}/kT} \int_0^\infty e^{-2\pi\eta} e^{-E_{01}/kT}\, dE_{01}$$

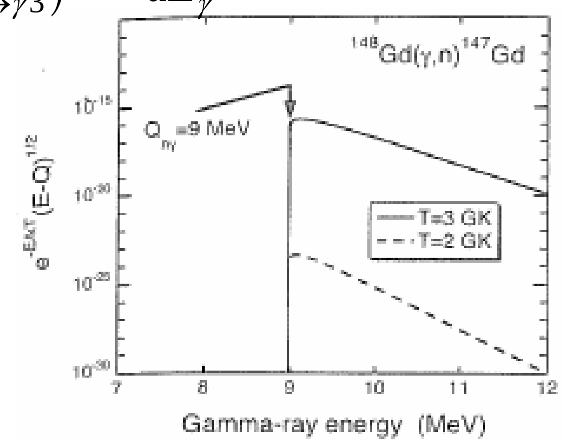
Again, the integrand in this equation represents the Gamow peak centered around. $E_\gamma^{eff} = E_0 + Q_{01\to\gamma 3}$

If we consider now reactions emitting neutrons with an energy smaller than the neutron binding energy we obtain the following equation:

$$\lambda_\gamma(3) \approx \int_0^\infty \left(E_\gamma - Q_{01\to\gamma 3}\right) e^{-E_\gamma/kT} E_{01}^{\ell-1/2}\, dE_\gamma \approx \int_0^\infty e^{-E_\gamma/kT} \left(E_\gamma - Q_{01\to\gamma 3}\right)^{\ell+1/2} dE_\gamma$$

In this case the energy window for (γ,n) reactions will be located at

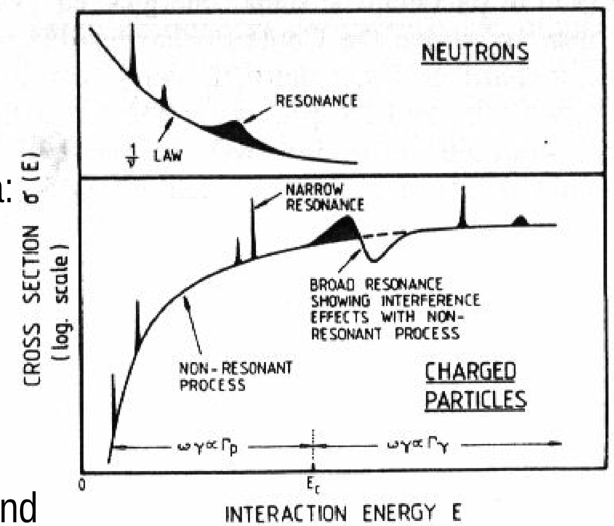
$$E_\gamma^{eff} = \left(\ell + 1/2\right)kT + Q_{n\gamma}$$



Resonant thermonuclear reactions

4.1 Narrow-resonance reaction rates

Possible resonances is the reaction cross sections could have an important impact on the reaction rates. If we consider narrow resonances (Γ less than few keV). Isolated resonances can be described by the Breit-Wigner formula:



$$\sigma_{BW}(E) = \frac{\lambda^2}{4\pi}\frac{(2J+1)(1+\delta_{01})}{(2j_0+1)(2j_1+1)}\frac{\Gamma_a\Gamma_b}{(E-E_r)^2+\Gamma^2/4}$$

where j_i are the spins of the target and projectile, J and E are the spin and energy of the resonance, Γ_i are the resonance partial widths of entrance and exit channel and Γ is the total resonances width and. $\lambda = 2\pi/k = 2\pi\hbar/\sqrt{2m_{01}E}$

The reaction rate for a single narrow resonance can then be obtained as:

$$N_A\langle\sigma v\rangle = \left(\frac{8}{\pi m_{01}}\right)^{1/2}\frac{N_A}{(kT)^{3/2}}\int_0^\infty E\sigma_{BW}(E)e^{-E/kT}dE = N_A\frac{\sqrt{2\pi}\hbar^2}{(m_{01}kT)^{3/2}}w\int_0^\infty\frac{\Gamma_a\Gamma_b}{(E-E_r)^2+\Gamma^2/4}e^{-E/kT}dE$$

where $\quad w = (2J+1)(1+\delta_{01})/[(2j_0+1)(1j_1+1)]$

Resonant thermonuclear reactions

4.1 Narrow-resonance reaction rates

For a sufficiently narrow resonance, the Maxwell-Boltzmann factor e-E/kT and the partial widths Gi are approximately constant over the total width of the resonance, then:

$$N_A\langle\sigma v\rangle = N_A \frac{\sqrt{2\pi}\hbar^2}{(m_{01}kT)^{3/2}} e^{-E_r/kT} w \frac{\Gamma_a\Gamma_b}{\Gamma} 2\int_0^\infty \frac{\Gamma/2}{(E-E_r)^2+\Gamma^2/4} dE = N_A \frac{\sqrt{2\pi}\hbar^2}{(m_{01}kT)^{3/2}} e^{-E_r/kT} w \frac{\Gamma_a\Gamma_b}{\Gamma} 2\pi$$

The area under the resonance can be obtained as:

$$\Gamma\cdot\sigma_{BW}(E=E_r) = \Gamma\cdot\frac{\lambda_r^2}{\pi} w \frac{\Gamma_a\Gamma_b}{\Gamma^2} = \frac{\lambda_r^2}{\pi}\omega\gamma \qquad \gamma = \frac{\Gamma_a\Gamma_b}{\Gamma^2}$$

The quantity $\omega\gamma$ is referred to as the *resonance strength*. According to these results the reaction rates depend Only on the energy and strength of the resonance, but not on the exact shape.

If several narrow and isolated resonances contribute to the cross section, then their contributions to the reaction rate add incoherently. Numerically one finds:

$$N_A\langle\sigma v\rangle = \frac{1.5399\cdot 10^{11}}{\left(\frac{M_0 M_1}{M_0+M_1}T_9\right)^{3/2}} \sum_i (\omega\gamma)_i e^{-11.605E_i/T_9} \qquad (cm^3 mol^{-1} s^{-1})$$

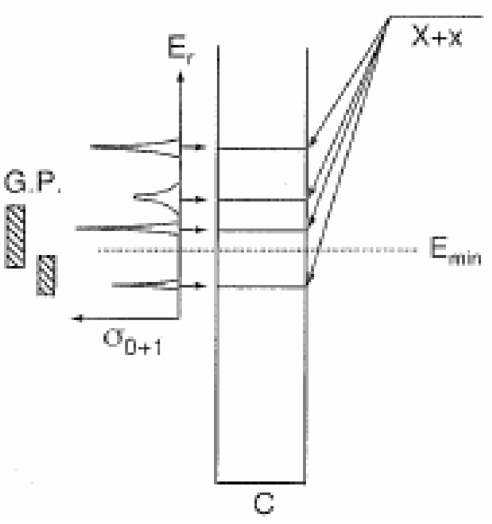
Resonant thermonuclear reactions

4.1 Narrow-resonance reaction rates

Narrow resonances in the range of effective stellar energies have a dramatic effect on reaction rates. Therefore, it is important to locate all narrow resonances that could contribute.

Generally one can measure this resonances down to an energy E_{min} representing the smallest cross section reachable in the laboratory because of the strong reduction of the cross sections with energy due to the Coulomb barrier.

Then one can use indirect methods to characterize the region between E=0 and $E=E_{min}$.One can uses other reactions X+x to populate astrophysically important levels in the compound nucleus C. From the measured nuclear properties of the Compound levels close to the particle threshold, the resonance energies and strengths of astrophysically relevant resonances can be estimated.



Resonant thermonuclear reactions

4.2 Broad-resonance reaction rates

When broad resonances are present the explicit energy dependence of the cross section is important. Let's consider the reaction ^{24}Mg(p,γ)^{25}Al with a broad resonance located at different energies:

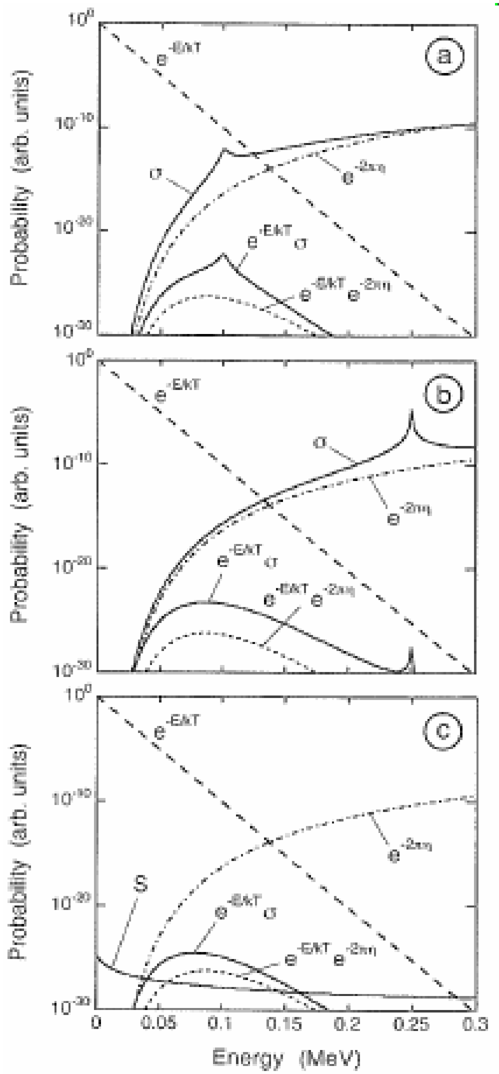
a) E_r=0.1 MeV, Γ=5 keV inside the Gamow peak. The energy dependence of the Maxwell-Boltzmann distribution and the reaction cross sections must be considered. Then the reaction rates have to be calculated numerically:

$$N_A\langle\sigma v\rangle = \sqrt{2\pi}\,\frac{N_A\omega\hbar^2}{(m_{01}kT)^{3/2}}\int_0^\infty e^{-E/kT}\,\frac{\Gamma_a(E)\Gamma_b(E+Q-E_f)}{(E-E_r)^2+\Gamma(E)^2/4}\,dE$$

where the exit channel Γ_b has to be calculated at the energy $E_{23}=E_{01-\to 23}-E_f$

b) E_r=0.25 MeV, Γ=6 keV above the Gamow peak. This case had no influence in the reaction rate for narrow resonances. However, now the product of the Maxwell-Boltzmann distribution and the cross sections gives rise to another maximun at low energies caused by the low-energy wing of the resonance.

c) Shows a subthreshold resonance, corresponding to a compound nucleus level located below the proton threshold. In this case the high-energy wing of the resonance affects the reaction rate.



5.1 Electron screening

In the fully ionized stellar plasma, the Coulomb potential between two interacting nuclei will be screened by the cloud of electrons surrounding them. The effective barrier for nuclear fusion reaction becomes thinner and, therefore, the tunneling probability and the reaction rate increase over their values obtained in vacuum. This effect is known as *electron screening*. The screened potential for two colliding nuclei *0* and *1* is given by:

$$V_s(r) = \frac{Z_0 Z_1 e^2}{r} e^{-r/R_D}$$

where R_D is the *Debye-Huckel radius* $R_D = \sqrt{\frac{kT}{4\pi e^2 \rho N_A \zeta^2}} = 2.812 \cdot 10^{-7} \rho^{-1/2} T_9^{1/2} \zeta^{-1} \quad (cm)$

and $\zeta = \sqrt{\sum_i \frac{\left(Z_i^2 + Z_i \vartheta_e\right) X_i}{A_i}}$ with θ_e being the electron degeneracy

The Debye-Huckel radius is a meadure of the size of the electron cloud. For most of the thermonuclear reactions this radius is much larger than the average distance between neighboring nuclei.

Taking into consideration this screening effect we can calculate a modified barrier transmission coefficient.

$$T \approx e^{x\pi\eta - 2\pi\eta}$$

$$x(E_0) = \left(R_c / R_D\right)_{E_0} = \frac{Z_0 Z_1 e^2}{E_0} \sqrt{\frac{4\pi e^2 \rho N_A}{kT}} \zeta = 4.197 \cdot 10^{-6} \left(Z_0 Z_1\right)^{1/3} \left(\frac{M_0 M_1}{M_0 + M_1}\right)^{-1/3} \sqrt{\rho} T_9^{-7/6} \zeta$$

$$\left(x\pi\eta\right)_{E_0} = \frac{Z_0 Z_1 e^2}{R_D} \frac{\left(\pi\eta\right)_{E_0}}{E_0} = \frac{Z_0 Z_1 e^2}{R_D kT} = 5.945 \cdot 10^{-6} \sqrt{\rho} Z_0 Z_1 T_9^{-3/2} \zeta$$

General aspects on thermonuclear reactions

5.1 Electron screening

Reaction rates can be calculated including the electron screening as:

$$N_A\langle\sigma v\rangle = \left(\frac{8}{\pi m_{01}}\right)^{1/2} \frac{N_A}{(kT)^{3/2}} \int_0^\infty S(E) e^{x\pi\eta} e^{-2\pi\eta} e^{-E/kT} dE$$

Although x and η depend on energy this expression can be approximated by evaluating the factor $e^{x\pi\eta}$ at the most effective energy of the interaction in the plasma which is the Gamow energy E_0.

$$N_A\langle\sigma v\rangle = \left(\frac{8}{\pi m_{01}}\right)^{1/2} \frac{N_A}{(kT)^{3/2}} f_s \int_0^\infty S(E) e^{-2\pi\eta} e^{-E/kT} dE$$

$$f_s = e^{(x\pi\eta)_{E_0}} = e^{Z_0 Z_1 e^2/(R_D kT)} = e^{5.945 \cdot 10^{-6}\sqrt{\rho} Z_0 Z_1 T_9^{-3/2}\zeta}$$

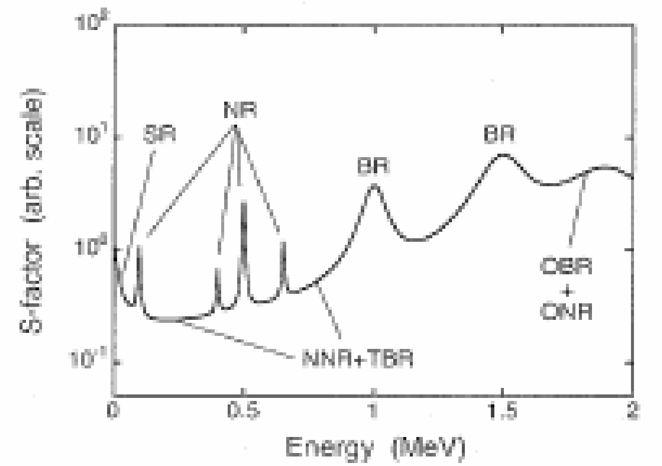
Consequently the screened reaction rate is simply obtained by multiplying the unscreened reaction rate by the screening factor f_s.

It should be also consider screening due to electrons of the target in laboratory measurements of nuclear reactions.

5.2 Total reaction rates

For the calculation of the total reaction rates, all processes contributing significantly to the reaction mechanism in the effective stellar energy range have to be taken into account.

$$N_A\langle\sigma v\rangle_{total} = \sum_i N_A\langle\sigma v\rangle^i_{\substack{narrow\\resonances}} + \sum_k N_A\langle\sigma v\rangle^k_{\substack{broad\\resonances}} + N_A\langle\sigma v\rangle_{nonresonant}$$



Decays in and ractions in stellar plasma

Exercise 1.
Taking into account the decay scheme of ^{26}Al into ^{26}Mg, determine the sellar half-life of ^{26}Al when the plasma temperature amounts to T=2 GK.

Exercise 2.
Determine the fraction of photons contributing to the reaction $^{26}Si(\gamma,p)^{25}Al$ in a stellar medium at temperature T=0.3 GK.

Exercise 3.
In a stellar plasma, the nucleus ^{25}Al may be destroyed by the capture reaction $^{25}Al(p,\gamma)^{26}Si$ or by β^+ decay ($T_{1/2}$=7.18 s). Neglecting other processes, determine the dominant destruction process at a stellar temperature of T=0.3GK assuming a reaction rate of $N_A(gv)=1.8\ 10^{-3}$ cm^{3}mol^{-1}s^{-1}, a stellar density of $\rho=10^4$ g/cm^3 and a hydrogen mass fraction of $X_H=0.7$.

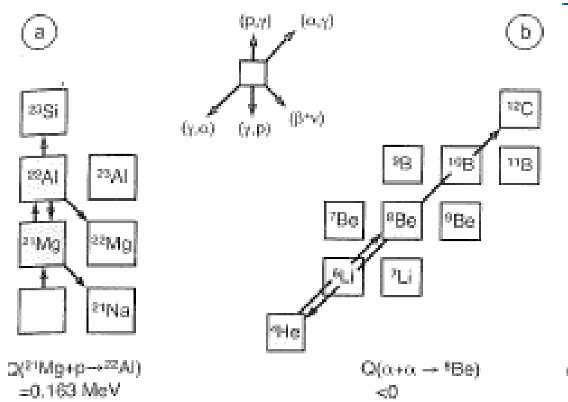
Exercise 4.
In a stellar plasma at 10 GK ^{32}S can be destroyed by radiative proton capture $^{32}S(p,\gamma)^{33}Cl$ with a reaction rate of 0.87 103 cm^{3}mol^{-1}s^{-1} (considering thermally excited states in ^{32}S). Determine the stellar rate for the reverse reaction by using the following partition functions $G_{32S}=1.6$, $G_p=1$ and $G_{33Cl}=1.9$.

Decays in and ractions in stellar plasma

Exercise 5.
In hydrogen burning enviroments at high temperatures the proton capture in ^{21}Mg produces ^{22}Al and this one again ^{23}Si. Considering that ^{21}Mg and ^{22}Al are in equilibrium, determine if the nucleosynthesis proceeds via ^{21}Mg β^+ decay or via sequential two-proton capture to ^{23}Si.

$$\lambda_{^{21}Mg \to ^{22}Al} = 1.1 \cdot 10^3 \text{ s}^{-1} \qquad \lambda_{^{21}Mg \to ^{21}Na} = 5.6 \text{ s}^{-1} \qquad \lambda_{^{22}Al \to ^{21}Mg} = 3.4 \cdot 10^7 \text{ s}^{-1}$$

$$\lambda_{^{22}Al \to ^{23}Si} = 3.1 \cdot 10^4 \text{ s}^{-1} \qquad \lambda_{^{22}Al \to ^{22}Mg} = 26.2 \text{ s}^{-1}$$



Exercise 6.
One of the most important proceeses in stellar mucleosynthesis in the triple-α reaction producing ^{12}C in two steps: $\alpha+\alpha \to {}^8Be$ and $\alpha + {}^8Be \to {}^{12}C$. Estimate the decay constant, $\lambda_{\alpha\alpha\alpha \to 12C}$ in a stellar medium at T=0.3 GK and density $\rho=10^5$ g/cm^3, assuming a mass fraction of $X\alpha=1$ and $N_A(\sigma v)_{\alpha+8Be\to 12C}=1.17\ 10^{-2}$ cm^3 mol^{-1} s^{-1}.