The language of a model category

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Abstract.

Quillen model categories are a cornerstone for modern homotopy theory. These categories, originally devised to capture homotopical properties of categories like topological spaces, simplicial sets or chain complexes, have gained relevance for giving a way to construe higher categories which are of great importance, for example, in algebraic topology and geometry.

In this talk, we will see that model categories also have logical information on their own in the following sense: Given any model category, we can associate to it a class of first-order formulas referring to the fibrant objects of the category. For example, the associated language of the category of small categories, equipped with its canonical model structure, coincides with language for categories defined by Blanc [1] and Freyd [2], whose central feature is that it respects the equivalence principle.

Similarly, the language we associate to a model category respects the appropriate version of the equivalence principle: two homotopically equivalent objects satisfy the same formulas and replacing parameters by homotopically equivalent ones does not change the validity of a formula.

Finally, we will show that for M and N two Quillen equivalent model categories, their associated languages are, suitably, equivalent.

References

[1] G. Blanc, *Equivalence naturelle et formules logiques en théorie des catégories*, Archiv für mathematische Logik und Grundlagenforschung 19 (1978), no. 1, 131–137.

[2] P. Freyd, *Properties invariant within equivalence types of categories*, Algebra, Topology and Category Theory, Academic Press   (1976), 55–61.