Two-dimensional limit theories enhanced

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Abstract.

Limit theories capture (many-sorted) sets with structure, including examples such as monoids, rings and small categories. Following Kelly [2], we can consider enriched limit theories for any suitable base of enrichment, so that when we enrich over small categories we obtain a notion of limit 2-theory, which ought to capture (many-sorted) categories with structure, such as monoidal categories, duoidal categories and double categories.

However, on a closer analysis, limit 2-theories do not not work as well as one might hope. The problems most clearly emerge in many-sorted examples of current interest such as monoidal fibrations, double categories, double fibrations and monoidal double categories where certain structural maps should be strict (and/or strictly preserved).

In this talk I will explain the problems with limit 2-theories and how they are overcome by passing to enhanced limit 2-theories, which are enriched limit theories over a different base of enrichment F [3]. Our main application is a theorem about bimodels in this setting and will explain the various equivalent descriptions of structures such as monoidal double categories [4] and double fibrations [1] in the literature.

References

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