Drazin Inverses in Categories

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Abstract.
   A Drazin inverse [1] is a fundamental algebraic gadget which has been extensively deployed in semigroup theory and ring theory. While they can also be defined for endomorphisms of any category, Drazin inverses (it seems) have never been extensively developed from a categorical perspective [4]. The purpose of this talk is to introduce Drazin inverses and to present some of their basic results. A Drazin category is a category in which every endomorphism has a Drazin inverse: examples include the category of matrices over a field, the category of finite length modules over any ring, and any finite set enriched category. We shall discuss Drazin inverses in mere categories, in additive categories, and in dagger categories. We shall explain how Drazin inverses relate to Leinster's notion of eventual image duality [2]. Finally we shall introduce a new notion of Drazin inverses for pairs of opposing maps, and as an application of this kind of Drazin inverse we provide, for dagger categories, a novel characterization of the Moore-Penrose inverse [3] in terms of the Drazin inverse of the opposing pair of a map and its adjoint.

References

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