Non-additive derived functors: a chain complex approach

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Abstract.

Let $F\colon \mathcal{C} \to \mathcal{E}$ be a functor from a category $\mathcal{C}$ to a (Borceux–Bourn [1]) homological or (Janelidze–Márki–Tholen [6]) semi-abelian category $\mathcal{E}$. We investigate conditions for the homology $\mathrm{H}_n(X, F)$ of an object X in $\mathcal{C}$ with coefficients in the functor F defined via projective resolutions in $\mathcal{C}$ to be independent of the chosen resolution. Then the left derived functors of F may be constructed as in the classical abelian case.

Our strategy is to extend the concept of *chain homotopy* to a non-additive setting via the technique of *imaginary morphisms*. More precisely, we use the *approximate subtractions* of Bourn–Janelidze [2], originally considered in the context of subtractive categories [7, 8]. This works as soon as $\mathcal{C}$ is a pointed regular category with finite coproducts and enough projectives which are *closed under protosplit subobjects*, a new condition we introduce in [3], and which comes for free in the abelian setting. We further assume that the functor F satisfies certain exactness conditions: we may ask it to be protoadditive [4, 5] and preserve binary coproducts and proper morphisms, for instance—conditions which amount to F being additive when $\mathcal{C}$ and $\mathcal{E}$ are abelian categories.

In this setting we work out a basic theory of derived functors, compare it with the simplicial approach, and give some examples.

The main reference of this talk is [3].

References

[1] F. Borceux and D. Bourn, *Mal'cev, protomodular, homological and semi-abelian categories*, Math. Appl., vol. 566, Kluwer Acad. Publ., 2004.

[2] D. Bourn and Z. Janelidze, *Subtractive categories and extended subtractions*, Appl. Categ. Structures 17 (2009), 317–343.

[3] M. Culot, F. Renault and T. Van der Linden, *Non-additive derived functors*, in preparation, 2024.

[4] T. Everaert and M. Gran, *Homology of n-fold groupoids*, Theory Appl. Categ. 23 (2010), no. 2, 22–41

[5] T. Everaert and M. Gran, *Protoadditive functors, derived torsion theories and homology*, J. Pure Appl. Algebra 219 (2015), no. 8, 3629–3676.

[6] G. Janelidze, L. Márki, and W. Tholen, *Semi-abelian categories*, J. Pure Appl. Algebra 168 (2002), no. 2–3, 367–386.

[7] Z. Janelidze, *Subtractive categories*, Appl. Categ. Structures 13 (2005), 343–350.

[8] A. Ursini, *On subtractive varieties I*, Algebra Universalis 31 (1994), 204–222.