The homotopy theory of Eilenberg-Zilber opetopic sets

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Abstract.

We present the construction of the category of Eilenberg-Zilber presheaves (EZ-presheaves for short), which is a coherent reflective subcategory of the category of presheaves on a Reedy category satisfying certain natural axioms (we call such Reedy category a tidy Reedy category).

This construction is motivated by the fact that the category of EZ-presheaves allows the use of techniques familiar from the theory of simplicial sets, such as induction over skeleta. In general, such techniques are not directly applicable to all presheaves. In fact, in the case of the category Δ, the category of EZ-presheaves is equal to the category of all presheaves, the fact which is not true for many other Reedy categories of interest. Consequently, some key properties of that construction are described.

Next, we apply our construction to the category $pOpe_\iota$ of positive opetopes with face maps and ι-contractions, introduced and studied by Zawadowski ([2]). As a preparation, we show that $pOpe_\iota$ is a tidy Reedy category. A minor modification of Olschok's theorem ([1]) allows us to endow this category $\widehat{pOpe_\iota}_{EZ}$ of opetopic EZ-presheaves with a model structure in a Cisinski style, which we call the opetopic $(\infty, 0)$-structure.

Finally, we construct two adjunctions between the categories $\widehat{pOpe_\iota}_{EZ}$ and $sSet$ and show that they are in fact Quillen equivalences (when the category of simplicial sets is considered with the Kan-Quillen model structure), by studying properties of certain fundamental opetopic sets, called opetopic associahedra (which are the images of representable simplicial sets in both of the adjunctions mentioned above).

References

[1] M. Olschok, *Left determined model structures for locally presentable categories*, Appl. Categ. Structures 19 (2011)

[2] M. Zawadowski, *Positive Opetopes with Contractions form a Test Category*, ArXiv:1712.06033 [math.CT], (2017).