Iterating semidirect products in semi-abelian categories

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Abstract.

In any semi-abelian category $\mathcal{C}$, the notions of semidirect product, internal action and split epimorphisms are equivalent [1]. In particular, this means that every split short exact sequence is, up to isomorphism, of the form

$$A \xrightarrow{j_A} A \rtimes B \underset{s_B}{\overset{p_B}{\rightleftarrows}} B.$$

Furthermore, the notion of action of B on A is internal, in the sense that it can be defined as a morphism $B \flat A \to A$ such that certain diagrams commute in $\mathcal{C}$. In the categories of groups and Lie algebras, a notion of n-semidirect products has been introduced by Carrasco and Cegarra [2, 3], which allows to construct certain iterated semi-direct products using a system of functions $A_k \times A_j \to A_i$ for $i \leq j < k$ satisfying certain identities.

In this talk we will explain how these n-semidirect products can be characterized by diagrams linking various exact sequences. We will also show how they can be constructed as colimits in $\mathcal{C}$, and how their structure can be made internal, i.e described by morphisms and commutative diagrams in $\mathcal{C}$, and how that description can be simplified when $\mathcal{C}$ is algebraically coherent [4].

We will also show how iterated semidirect products can be related to iterated internal actions, and use this relation to explain when the iteration is associative.

References

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