Gray multicategories and left and right Gray skew-multicategories

B. Femić

Bojana Femić (femicenelsur@gmail.com)
Mathematical Institute of Serbian Academy of Sciences and Arts

Abstract.

Let $\mathcal{O}_n$ be the category $(n\text{-}\mathrm{Cat})\text{-}\mathrm{Cat}$ of categories enriched in n-categories, or $\mathrm{Cat}(n\text{-}\mathrm{Cat})$, the category of categories internal in n-categories, with $n \geq 1$. We introduce *Gray multicategory* and *left and right Gray skew-multicategory* of $\mathcal{O}_n$. They all differ by *type* $(*, \bullet)$ which depends on the nature $*$ of functors and $\bullet$ of transformations used in their construction. We show (in law dimensions) that left Gray skew-multicategories of certain $\bullet_1$-type are left closed and left representable, whereas right Gray skew-multicategories of other $\bullet_2$-type are right closed and right representable. For Gray multicategories we show that: 1) those of strict $*$-type are representable, and 2) that those of certain $\bullet_1$-type are left closed, and those of other $\bullet_2$-type are right closed, both for any $*$-type, and that they are related by duality. Using the results of Hermida on the equivalence of representable multicategories and monoidal categories, and of Bourke-Lack that left representable skew-multicategories yield skew-monoidal categories, we obtain non-Cartesian monoidal and skew-monoidal (closed) categories of internal and enriched categories depending on the type. Our construction generalizes results of Gray and Böhm of closedness and Gray-monoidality for the categories of 2-categories and double categories (in the sense of dimension and type, respectively), and recent results of Bourke-Lobbia for Gray-categories (in the sense of type). The approach we use relies on the tools developed in [4] and [5].

This is a work in progress, the results are expected not to depend on dimension n.

References

[1] G. Böhm, *The Gray Monoidal Product of Double Categories*, Appl. Categ. Structures 28 (2020), 477–515.

[2] J. Bourke, S. Lack, *Skew monoidal categories and skew multicategories*, J. Algebra 56 (2018), 237–266.

[3] J. Bourke, G. Lobbia: *A skew approach to enrichment for Gray-categories*, Adv. Math. 434 (2023).

[4] B. Femić: *Bifunctor Theorem and strictification tensor product for double categories with lax double functors*, Theory Appl. Categ. 39 (2023), no. 29, 824–873.

[5] B. Femić, *Enrichment and internalization in tricategories, the case of tensor categories and alternative notion to intercategories*, Filomat 38, (2004), no. 8, 2601–2660.

[6] J. W. Gray, *Formal category theory: adjointness for 2-categories*, Lecture Notes in Mathematics, vol. 391, Springer-Verlag, Berlin-New York (1974) 1, 19, 27.

[7] C. Hermida, *Representable multicategories*, Adv. Math. 151 (2000), no. 2, 164–225.