Universal central extensions of braided crossed modules of groups

A. Fernández Fariña

José Manuel Casas (jmcasas@uvigo.es)
Universidade de Vigo & CITMAga

Alejandro Fernández Fariña (alejandrofernandez.farina@usc.es)
Universidade de Santiago de Compostela

Manuel Ladra (manuel.ladra@usc.es)
Universidade de Santiago de Compostela & CITMAga

Abstract.

The concept of central extension of groups is highly relevant in mathematics, for instance, in the interpretation of the third cohomology, and it plays a fundamental role in several areas of physics as well in the quantization of symmetries.

Crossed modules of groups are algebraic objects equivalent to strict 2-groups, or equivalently categorical groups. Since crossed modules of groups are a generalization of groups, it is natural to search extensions of classical results in the theory of groups in the category of crossed modules of groups, both examples of semi-abelian categories [3].

Joyal and Street defined in [4] the concept of braiding for monoidal categories as a natural isomorphism $\tau_{X,Y} \colon X \otimes Y \to Y \otimes X$, generalizing the idea of the usual tensor product of vector spaces. The notion of braiding for categorical groups provides an equivalent category to the category of braided crossed modules of groups (see [2, 4]).

In this talk, we will devise a braided version of the results given by Norrie in [5] for braided crossed modules of groups in the framework of a semi-abelian category (see[1]); more precisely, we will study universal central extensions in the category of braided crossed modules.

For that purpose, we will construct the universal central extension of a braided crossed module in the category of braided crossed modules. Then, we will also give a canonical braiding on the universal central extension of a crossed module with a given braiding in the category of crossed modules. To finish, we will show the relationship between the two constructions.

References

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