An Algebraic Combinatorial Approach to the Abstract Syntax of Opetopic Structures

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Abstract.

The starting point of the talk will be the identification of structure common to tree-like combinatorial objects, exemplifying the situation with abstract syntax trees (as used in formal languages) and with opetopes (as used in higher-dimensional algebra). The emerging mathematical structure will be then formalised in a categorical setting, unifying the algebraic aspects of the theory of abstract syntax [2, 3] and the theory of opetopes [6]. This realization allows one to transport viewpoints between the two mathematical theories and I will explore it here in the direction of higher-dimensional algebra, giving an algebraic combinatorial framework for a generalisation of the slice construction [1] for generating opetopes. The technical work will involve setting up a microcosm principle for near-semirings [5] and exploiting it in the bicategory of generalised species of structures [4], the cartesian closed structure of which plays a fundamental role.

References

[1] J. Baez and J. Dolan. Higher-Dimensional Algebra III. n-Categories and the Algebra of Opetopes. Advances in Mathematics 135(2):145–206, 1998.

[2] M. Fiore, G. Plotkin and D. Turi. Abstract syntax and variable binding. In Proceedings of the 14th Annual IEEE Symposium on Logic in Computer Science (LICS'99), pages 193–202. IEEE, Computer Society Press, 1999.

[3] M. Fiore. Second-order and dependently-sorted abstract syntax. In Proceedings of the 23rd Annual IEEE Symposium on Logic in Computer Science (LICS'08), pages 57–68. IEEE, Computer Society Press, 2008.

[4] M. Fiore, N. Gambino, M. Hyland, and G. Winskel. The cartesian closed bicategory of generalised species of structures. J. London Math. Soc., 77:203–220, 2008.

[5] M. Fiore and P. Saville. List objects with algebraic structure. In Proceedings of the 2nd International Conference on Formal Structures for Computation and Deduction (FSCD 2017), No. 16, pages 1–18, 2017.

[6] S. Szawiel and M. Zawadowski. The web monoid and opetopic sets. Journal of Pure and Applied Algebra, 217:1105–1140, 2013.