Monoidal Meta-Theorem

D. Forsman

David Forsman (david.forsman@uclouvain.be)
Université catholique de Louvain

Abstract.

Certain families of theories of multi-sorted universal algebra can be modeled in monoidal, symmetric monoidal, and cartesian monoidal categories, respectively. For each of these families of theories, we produce a sound deduction system $\vdash$. We show that these deduction systems are complete with respect to the cartesian monoidal category of sets. This yields a meta-theorem:

Let C be a (cartesian/symmetric) monoidal category and let $E \cup \{\phi\}$ be a (cartesian/symmetric) monoidal theory of universal algebra. Then

$$E \models \phi \textit{ in } \mathbf{Set} \textit{ implies that } E \models \phi \textit{ in } C.$$

The Monoidal Meta-Theorem makes a modest connection between the algebraic structures in **Set** to monoidally enriched algebraic structures. As a corollary, we attain that the Eckmann-Hilton argument generalizes to the setting of symmetric monoidal categories.

The meta-theorem for cartesian monoidal categories is essentially known and proven in [1].

References

[1] P. T. Johnstone, *Sketches of an Elephant: A Topos Theory Compendium: Volume 2*, Oxford Logic Guides, Clarendon Press, 2002.