Conditional flatness and fiberwise localizations in semi-abelian categories

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Abstract. In [1] we extended the group-theoretic notion of *conditional flatness* for a localization functor to any pointed category, and we investigated it in the context of homological and semi-abelian categories. This context includes several examples of importance in algebra, such as groups, loops, Lie algebras, crossed modules, C*-algebras, etc. In the presence of *functorial fiberwise localization*, many results analogous to those obtained in the category of groups [2] hold in any semi-abelian category, and we provide some existence theorems for certain localization functors in some specific categories, such as groups, compact groups and cocommutative Hopf algebras. Among reflective subcategories, the so-called *Birkhoff subcategories* are the ones that are also closed in the larger category under subobjects and quotients. We prove that any Birkhoff subcategory of a semi-abelian category provides a conditionally flat localization, and explain how the property of conditional flatness of a functor actually corresponds to the property of admissibility of an adjunction from the point of view of categorical Galois theory [3].

We then present a new example of Birkhoff subcategory [4]. When $\mathbb{C}$ is a regular Mal'tsev category, hence in particular if $\mathbb{C}$ is semi-abelian, the category $2\text{-}\mathsf{Grpd}(\mathbb{C})$ of internal 2-groupoids in $\mathbb{C}$ can be shown to be a Birkhoff subcategory of the category $\mathsf{Grpd}^2(\mathbb{C})$ of double groupoids in $\mathbb{C}$, and a simple description of the reflector can be given. Under some natural conditions on a semi-abelian category $\mathbb{C}$ first considered in [5], the semi-abelian category $2\text{-}\mathsf{Grpd}(\mathbb{C})$ turns out to be also action representable in the sense of [6].

These results have been obtained in collaboration with Jérôme Scherer [1] and Nadja Egner [4].

References

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