# From Kripke models to neighborhood models in category theory

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**Abstract.**
There is a well-known one-to-one correspondence between Kripke models and augmented neighbourhood models, where an augmented model is a monotonic neighbourhood model which contains its core ($\bigcap N(w) \in N(w)$ for all $w$), (see [4], original proof in [1]). Axiom K is valid in Kripke models, but not in neighborhood models in general. If we consider the two categories $\mathcal{KM}$ and $\mathcal{NM}$, we can set an injective functor $f : \mathcal{KM} \to \mathcal{NM}$ that sends every Kripke model to a modally equivalent (augmented) neighbourhood model; alternatively, $\mathcal{KM}$ is isomorphic to the subcategory of augmented neighbourhood models, in which axiom K is valid. If we consider the quotient categories of bisimilar Kripke models $\mathcal{KM}\backslash bisim$ and neighbourhood models $\mathcal{NM}\backslash bisim$, we can establish an analogue functor $f'$ between them preserving modal equivalence. Considering the injective functors $i_k : \mathcal{KM} \to \mathcal{KM}\backslash bisim$ and $i_n : \mathcal{NM} \to \mathcal{NM}\backslash bisim$, we have that $i_n \circ f = f' \circ i_k$. This can also be studied among specific kinds of models (reflexive, transitive, and so on).

Coalgebras over the category *Set* allow us to abstract transition structures like Kripke and neighbourhood frames and models, and also other structures such as labelled transition systems and deterministic automata (see [3], [5]). Thus, the aforementioned relationship among Kripke and neighborhood models can be studied in a more general perspective. Coalgebras offer an abstract framework that can be applied to generalise well-known notions from Kripke frames and models such as bisimilarity or image-finiteness for broader families of neighbourhood models and frames [2].

## References

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