A relative comonad associated to the category of partial comodules

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Abstract. Partial comodules over a Hopf algebra can be seen as a generalization of usual comodules: the coaction should no longer be coassociative, but only *partially* coassociative. This setting is dual to the one of partial modules over a Hopf algebra, which were introduced in [2] and generalize partial actions of groups to Hopf algebras.

The category of partial comodules does not satisfy the so-called fundamental theorem of comodules: a partial comodule is not guaranteed to be equal to the sum of its finite-dimensional subcomodules. Furthermore, in general there does not even exist a coalgebra whose category of comodules is equivalent to the category of partial comodules. This is in contrast to the dual theory of partial modules; it is known that the category of partial modules over a Hopf algebra is equivalent to the category of modules over a suitably constructed algebra which moreover has the structure of a Hopf algebroid. However, partial comodules are comonadic over vector spaces, as was shown in [3].

In order to better study the structure of this comonad (e. g., could it be lifted to a Hopf monad on some monoidal category?), it is useful to look at an associated *relative comonad*. The notion of relative comonad is dual to relative monads as introduced in [1]. In our case, the comonad is relative to the inclusion functor of vector spaces into complete topological vector spaces, and it induces a comonad on this last category.

References

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