Formalizing accessibility and duality in a virtual equipment

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Abstract. Gabriel–Ulmer duality is a duality between logical theories and categories of their models. The categories of models are called locally presentable categories and are known to be characterized as ind-completions of theories. Accessible categories are a generalization of locally presentable categories and are also characterized as ind-completions. In the accessible context, there are many known "Gabriel–Ulmer-like" dualities, for example, Makkai–Paré duality [4], Adamek–Lawvere–Rosický duality [1], and so on. Even in the enriched accessible context, there are many kinds of duality [5]. Then, an axiomatic approach to these dualities is suggested by [3]. That approach works in a 2-category and characterizes ind-completions, which are the core concept of dualities, as a KZ-monad.

In this talk, we will give another axiomatic approach to accessibility and duality in a double-categorical setting. More precisely, inspired by [2], we will work in an (augmented) virtual equipment rather than a 2-category. We characterize an ind-completion as a vertical morphism having a "Yoneda-like" universal property, which we call an *ind-morphism*. Then, we will show that the ind-morphisms form a double-categorical counterpart of relative (bi)adjunctions and that it yields a duality theorem. This talk is based on joint work with Keisuke Hoshino.

References

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