The universal property of the decomposition space of quasisymmetric functions

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Abstract. The coalgebra **QSym** of quasisymmetric functions was shown by Aguiar, Bergeron, and Sottile [1] to be the terminal object in the category of graded coalgebras with a zeta function. I'll explain an objective version of that result: **QSym** is the incidence coalgebra of a decomposition space Q of monotone surjections, and its zeta function Z is given by the empty surjection and the connected surjections [2]. We show that for any graded decomposition space X with a zeta function F, there is a unique graded span of decomposition spaces

$$X \xleftarrow{\alpha} J \xrightarrow{\varphi} Q$$

where α is ikeo (inner Kan and equivalence on objects) and φ is culf (conservative and unique lifting of factorisations) inducing F from Z. (Such spans induce coalgebra homomorphisms, and conjecturally all.)

This is joint work with Philip Hackney and Jan Steinebrunner [3].

References

[1] Marcelo Aguiar, Nantel Bergeron, and Frank Sottile. *Combinatorial Hopf algebras and generalized Dehn-Sommerville relations.* Compos. Math. **142** (2006), 1–30.

[2] Imma Gálvez-Carrillo, Joachim Kock, and Andrew Tonks. *Decomposition spaces of quasi-symmetric functions.* Unpublished.

[3] Philip Hackney, Joachim Kock, and Jan Steinebrunner. *The universal property of the decomposition space of quasisymmetric functions.* In preparation.