Formal Day convolution and low-dimensional monoidal fibrations

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Abstract.

Let T be a monad on an augmented virtual double category $\mathcal{K}$, the latter in the sense of [1]. The main result of this talk describes conditions ensuring that a formal Yoneda embedding $y\colon A \to P$ in $\mathcal{K}$ (in the sense of [2]) can be lifted along the forgetful functor $U\colon \mathsf{Lax}{-}T{-}\mathsf{Alg} \to \mathcal{K}$, where $\mathsf{Lax}{-}T{-}\mathsf{Alg}$ is the augmented virtual double category of lax T-algebras.

Taking $\mathcal{K} = \mathsf{Prof}$ the augmented virtual double category of profunctors and T the "free strict monoidal category"-monad the main result recovers the Day convolution monoidal structure on the category of presheaves $P = \mathsf{Set}^{A^{\mathrm{op}}}$ on a monoidal category A. Taking the same monad on the augmented virtual double category $\mathcal{K} = \mathsf{dFib}$ of two-sided discrete fibrations instead, the main result implies the "monoidal Grothendieck equivalence" of lax monoidal functors $A \to \mathsf{Set}$ and monoidal discrete opfibrations with base A (a variation on a result in [3] by Moeller and Vasilakopoulou).

Moving up a dimension, given a 2-monoidal 2-category A the main result likewise implies the equivalence of lax 2-monoidal 2-functors $A \to \mathsf{Cat}$ and 2-monoidal locally discrete split 2-opfibrations with base A. The main ingredient here is that (somewhat surprisingly) there exists an augmented virtual double category that accommodates the lax natural transformations required to define the formal Yoneda embedding induced by the Grothendieck equivalence for locally discrete split 2-opfibrations (the latter obtained by Buckley [4] and Lambert [5]).

I will report on work in progress on "internalising" the equivalence for 2-monoidal locally discrete split 2-opfibrations described above, thus obtaining an analogous equivalence for monoidal double split opfibrations (double fibrations in the sense of Cruttwell, Lambert, Pronk and Szyld [6]).

References

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