Poc sets and median algebras: A categorical duality

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Abstract.

The construction of a monad from an adjunction has a generalization allowing certain kinds of functors to induce monads as well. In particular, a functor whose target is rich enough in limits induces a codensity monad. In the presence of a left adjoint, the codensity monad of a functor agrees with the corresponding adjunction-induced monad. Since inclusion functors associated to subcategories of finite objects are unlikely to admit a left adjoint, their codensity monads are of interest. Familiar categories to consider are sets or vector spaces. In these cases, we get the ultrafilter and double dualization monads [1]. The goal of this talk is to compute a dual pair of codensity monads analogous to those just mentioned.

An important topic in geometric group theory is the study of group actions on trees. For this purpose, two vast generalizations of trees arose independently: poc sets and median algebras. Boolean algebras are an example of both, and in fact any set that simultaneously has suitably compatible structures of a poc set and median algebra is a Boolean algebra. The two-element Boolean algebra represents a pair of dualization functors between the categories of poc sets and median algebras. The dual of a poc set is its median algebra of ultrafilters, and the dual of a median algebra is its poc set of halfspaces [2]. In this talk, I will demonstrate that this pair of dualization functors exhibits similar properties to the dualization functor on vector spaces. I will show how dualization fits into a two-variable tensor-hom adjunction, and that both double dualization monads are the codensity monads of inclusions from finite objects of each category. Time permitting, I will say more about the kinds of Boolean algebras one can construct as the tensor product of a poc set and a median algebra.

References

[1] T. Leinster, *Codensity and the ultrafilter monad*, Theory Appl. Categ. 28 (2013), No. 13, 332-370

[2] M.A. Roller,*Poc sets, median algebras and group actions. An extended study of Dunwoody's construction and Sageev's theorem*, Preprint, Univ. of Southampton, 1998.