Bicategories for automata theory

F. Loregian

Fosco Loregian (fosco.loregian@taltech.ee)
Tallinn University of Technology

Abstract.

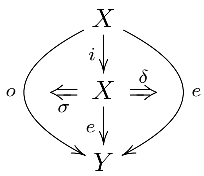
It has long been known [EKKK74] that automata can be interpreted within every monoidal category $(\mathcal{K}, \otimes, I)$; the cornerstone results in this direction are essentially three:

S1. if $T : \mathcal{K} \to \mathcal{K}$ is a commutative monad, 'Mealy' and 'Moore' machines in the (monoidal) Kleisli category $\mathcal{K}_T$ are 'non-deterministic' machines for a notion of fuzziness prescribed by T (examples of this are: exceptions monads, various probability monads, the powerset monad);

S2. if $\mathcal{K}$ is closed, one can characterize Mealy and Moore machines coalgebraically [Jac06], and this provides a slick proof of the cocompleteness of the categories $\mathbf{Mly}(A, B)$ and $\mathbf{Mre}(A, B)$ that they form [AT90];

S3. if (and curiously enough, *only if*) $\mathcal{K}$ is Cartesian monoidal, $\mathbf{Mly}(A, B)$ is the hom-category of a bicategory **Mly** [Gui74, KSW97], and $\mathbf{Mre}(A, B)$ the hom-category of a *semibicategory* (a bicategory without identity 1-cells, cf. [Mit72, MBCB02] and [BFL⁺23]) **Mre**.

Starting from the well-known principle that regards a monoidal category as nothing but a single-object bicategory, we fix a general bicategory $\mathcal{B}$ and study 'abstract machines' in $\mathcal{B}$, i.e. diagrams of 2-cells of the form



where i, e, o are 1-cells respectively dubbed the 'input' 1-cell, the 'state' 1-cell and the 'output' 1-cell.

We then proceed to find parallels for S1, S2, S3 in this more general setting:

B1. let T be a monad on **Set** and $(V, \odot, \perp)$ a quantale. The study of bicategorical machines in the bicategory of (T, V)-relations of [HST14] accounts for notions of non-determinism that are modeled on topologies, approach structures, metric and ultrametric structures, Kuratowski closure spaces, and all the likes of structures studied by monoidal topology;

B2. in perfect parallel with the monoidal case, the *behaviour* of a Mealy/Moore machine can be characterized through a universal property [Gog72]; a terminal coalgebra for monoidal machines, a weighted limit of sorts for bicategorical machines. In the case of Moore machines the description is prettier, in terms of a (pointwise) right extension. This clarifies long-forgotten remarks by Bainbridge [Bai75] on properties of abstract machines seen as Kan extensions;

B3. passing from single- to multi-object bicategories, we gain an additional degree of freedom by indexing hom-categories over generic objects; in particular, we gain a rich compositional structure that was not present in the monoidal case, a way of composing machines that is neither sequential nor parallel and that we dub *intertwining*.

This talk presents, and expands on, joint works with A. Laretto, G. Boccali, B. Femić, S. Luneia, see [BLLL23, BFL+23]

References

[AT90] J. Adámek and V. Trnková, *Automata and algebras in categories*, Kluwer, 1990.

[Bai75] E.S. Bainbridge, *Addressed machines and duality*, Category Theory Applied to Computation and Control (Berlin, Heidelberg) (Ernest Gene Manes, ed.), Springer Berlin Heidelberg, 1975, pp. 93–98.

[BFL+23] G. Boccali, B. Femić, A. Laretto, F. Loregian, and S. Luneia, *The semibicategory of moore automata*, 2023.

[BLLL23] Guido Boccali, Andrea Laretto, Fosco Loregian, and Stefano Luneia, *Bicategories of automata, automata in bicategories*, Electronic Proceedings in Theoretical Computer Science **397** (2023), 1–19.

[EKKK74] H. Ehrig, K.-D. Kiermeier, H.-J. Kreowski, and W. Kühnel, *Universal theory of automata. A categorical approach*, Teubner Studienbücher. Informatik. Stuttgart, 1974 (English).

[Gog72] J. Goguen, *Realization is universal*, Mathematical systems theory **6** (1972), 359–374.

[Gui74] R. Guitart, *Remarques sur les machines et les structures*, Cahiers de Topologie et Géométrie Différentielle Catégoriques **15** (1974), 113–144.

[HST14] D. Hofmann, G.J. Seal, and W. Tholen, *Monoidal topology: A categorical approach to order, metric, and topology*, vol. 153, Cambridge University Press, 2014.

[Jac06] B. Jacobs, *A bialgebraic review of deterministic automata, regular expressions and languages*, Algebra, Meaning, and Computation, Springer Berlin Heidelberg, 2006, pp. 375–404.

[KSW97] P. Katis, N. Sabadini, and R.F.C. Walters, *Bicategories of processes*, Journal of Pure and Applied Algebra **115** (1997), no. 2, 141–178.

[MBCB02] M.-A. Moens, U. Berni-Canani, and F. Borceux, *On regular presheaves and regular semicategories*, Cahiers de Topologie et Géométrie Différentielle Catégoriques **43** (2002), no. 3, 163–190 (en). MR 1928230

[Mit72] B. Mitchell, *The dominion of isbell*, Transactions of the American Mathematical Society **167** (1972), no. 0, 319–331.