Weak equivalences between algebraic weak ω-categories

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Abstract.

A *weak ω-category* in Leinster's sense [1] (see also Batanin [2]) has "all" the operations that a strict ω-category has, but need not satisfy any of the relations. In this setup, one can generalise certain arguments concerning strict ω-categories to weak ones by *encoding relations as operations*.

For example, consider the *unit law*. Given a 1-cell $f\colon x \to y$ in a weak ω-category, we can make sense of the expression $f \circ 1_x$ (because we have all the operations, including identities and compositions), but the relation $f \circ 1_x = f$ does not necessarily hold. Instead, we can consider the operation in a strict ω-category that takes a 1-cell f and spits out a(n identity) 2-cell $f \circ 1_x \to f$. This operation can be lifted to the weak ω-category, and (using a result [3] presented at CT2023) one can check that the resulting 2-cell is invertible in a suitable sense, establishing a kind of unit law.

In this talk, I will describe how to make use of such encoding and prove that the class of *weak equivalences* (an ω-dimensional version of essentially surjective, fully faithful functors) enjoys the *2-out-of-3 property*, i.e. if any two of F, G and GF are weak equivalences then so is the third, generalising the strict case treated in [4].

References

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