Internalization of decorated bicategories via π_2-indexings

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Abstract.

Given a bicategory $\mathcal{B}$, and a category $\mathcal{B}^*$, such that the collections of objects of $\mathcal{B}$ and $\mathcal{B}^*$ are equal, we wish to construct interesting double categories $\mathbb{D}$ having $\mathcal{B}$ as horizontal bicategory, and having $\mathcal{B}^*$ as category of objects. We say that the pair $(\mathcal{B}^*, \mathcal{B})$ is a decorated bicategory and $\mathbb{D}$ is an internalization of $(\mathcal{B}^*, \mathcal{B})$. The problem of understanding internalizations of decorated bicategories has been considered in the series of papers [1, 2, 3], where the definition of a numerical invariant, called the vertical length $\ell\mathbb{D}$, associated to every double category $\mathbb{D}$, was introduced. Roughly, the number $\ell\mathbb{D}$ measures the amount of work one would be expected to do to construct a generic square in $\mathbb{D}$, from squares in $(\mathbb{D}_0, H\mathbb{D})$. 1 is the minimum possible length of a double category, and most double categories in the literature, e.g. $\mathbb{M}\mathbf{od}, \mathbb{P}\mathbf{rof}, \mathbb{B}\mathbf{ord}, \mathbb{A}\mathbf{dj}$ are of length 1.

The particual problem of deciding whether a decorated bicategory $(\mathcal{B}^*, \mathcal{B})$ admits internalizations of length 1 has been study in [4]. We present a type of structure allowing to construct internalizations of lenght 1. We call the structure we study π_2-indexings, which are a type of indexing associated to a decorated bicategory $(\mathcal{B}^*, \mathcal{B})$, relating the arrows of $\mathcal{B}^*$ with 2-cells of a specific type in $\mathcal{B}$. The goal of the talk is to present the main results in [4], examples and conjectures related to the problem.

References

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