Opposites and hom weak ω-categories

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Abstract. This talk is based on our recent preprint [1]. We work with globular weak ω-categories, using a recent formulation for them and their computads proposed by Dean et al. [3]. We define opposites of a weak ω-category, changing the direction of all cells whose dimension belongs to a given set. We also give an alternative construction of the hom ω-categories of an ω-category to that of Cottrell and Fujii [2]. We then show that it has an left adjoint and that it preserves the property of being cofibrant.

Computads are structures out of which one can generate weak ω-categories. They consist of sets of generators together with attachment maps, assigning a source and target to each generator. In Dean et al. [3], first, the category Comp of computads and morphisms of ω-categories is defined inductively on dimension, together with an adjunction with the category Glob of globular sets

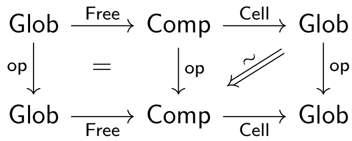
$$\mathsf{Glob} \underset{\mathsf{Cell}}{\overset{\mathsf{Free}}{\underset{\longleftarrow}{\overset{\longrightarrow}{\bot}}}} \mathsf{Comp} \ .$$

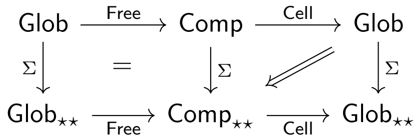
Here, the functor Cell takes a computad to the underlying globular set of the ω-category that it generates, the elements of which are either generators, or formal applicatons of operations of weak ω-categories (compositions and coherences). Then ω-categories are defined as the algebras for the induced monad on globular sets, which was shown to agree with that of Batanin and Leinster [5]. It was further shown by Garner [4] that ω-categories generated by a computad are the cofibrant objects for certain weak factorisation system.

We use a similar technique to construct the opposites and the homs of an ω-category. We start with an adjunction on the level of globular sets:

$$\mathsf{Glob} \underset{\mathsf{op}}{\overset{\mathsf{op}}{\underset{\longleftarrow}{\overset{\longrightarrow}{\bot}}}} \mathsf{Glob} \qquad\qquad \mathsf{Glob} \underset{\Omega}{\overset{\Sigma}{\underset{\longleftarrow}{\overset{\longrightarrow}{\bot}}}} \mathsf{Glob}_{\star\star}$$

where $\mathsf{Glob}_{\star\star}$ is the category of globular sets with two chosen objects. The functor op is defined with respect to a set of dimensions $w \subset \mathbb{N}_{>0}$, by swapping the source and target of every element whose dimension belongs to w. The functor Ω takes a globular set with two chosen objects to the globular sets of elements between those objects, and the suspension ΣX is the globular set with two objects, such that $\Omega(\Sigma X) = X$. We then observe that in both cases the left adjoint preserves the globular pasting diagrams, which are the arities of the operations of weak ω-categories. Using this observation, we define the opposite and the suspension of a computad together with natural transformations, as shown in the diagrams below:





The mates of those natural transformations under the respective adjunctions give rise to morphisms of monads, hence they lift as functors $\mathsf{op} : \omega\,\mathsf{Cat} \to \omega\,\mathsf{Cat}$ and $\Omega : \omega\,\mathsf{Cat}_{\star\star} \to \omega\,\mathsf{Cat}$. The functor op is an equivalence of categories with itself as its inverse, and by the adjoint lifting theorem, the functor Ω admits a left adjoint Σ.

Finally, we show that the functors op, Ω and Σ preserve the cofibrant objects, by describing a recognition principle for free ω-categories on a computad. We also show that the opposite of a hom ω-category is the hom of the some opposite of the original ω-category, as expected.

References

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[3] C. Dean, E. Finster, I. Markakis, D. Reutter and J. Vicary, Computads for weak ω-categories as an inductive type, preprint arXiv:2208.08719, 2022.

[4] R. Garner, *Homomorphisms of higher categories*, Advances in Mathematics, 2010, 224(6):2269-2311. doi:10.1016/j.aim.2010.01.022.

[5] T. Leinster, *Higher Operads, Higher Categories*, Cambridge University Press, 2004.