Eilenberg-Moore Bicategories for Opmonoidal Pseudomonads

A. Miranda

Adrian Miranda (adrian.miranda@manchester.ac.uk)
University of Manchester

Abstract. Given an monad T on a strict monoidal category $(\mathcal{V}, \otimes, I)$, suitably coherent natural transformations with components of the form $\chi_{X,Y} : T(X \otimes Y) \to TX \otimes TY$ and $\iota : TI \to I$ comprise a structure known as an opmonoidal monad. In the presence of such structure, the category of Eilenberg-Moore algebras $\mathcal{V}^T$ for the underlying monad also inherits a monoidal structure [6, 8]. Similarly, braidings and symmetries also lift to categories of algebras under suitable compatibility conditions. These results follow from two-dimensional monad theory, specifically the theory of Eilenberg-Moore objects in 2-categories of strict algebras and lax morphisms [4]. Alternatively, they also follow from the observation that the Eilenberg-Moore construction and products are both limits, and hence commute with one another, and opmonoidal monads are monoids in the 2-category of monads [10, 11].

In this talk I will discuss how these results extend to the two-dimensional setting. In this setting, the Eilenberg-Moore construction for pseudomonads is still a limit [3], however the theory of limits for lax morphisms of algebras for three-dimensional monads is far more complicated and not as well-developed [9]. Moreover, the appropriate monoidal structures on 2-categories [1] are now monoids in a non-cartesian monoidal structure, and as such monoidality of the Eilenberg-Moore construction for pseudomonads needs to be checked directly. Indeed, an important stepping stone is to check that the **Gray**-tensor product actually extends to pseudomonads. Once we have done all of this we find that the 2-category of pseudoalgebras $\mathcal{V}^T$ [5] inherits a monoidal structure that is slightly weaker than the original structure on $\mathcal{V}$, with associativity and unit laws holding up to 2-natural isomorphisms which satisfy the usual monoidal category axioms on the nose. We also describe similar liftings to pseudoalgebras for braidings, syllapses and symmetries that are suitably compatible with the pseudomonad structure.

This talk is based on results in [7]. Motivating applications include two-dimensional linear algebra and bicategorical models of linear logic [2]. This research is supported by EPSRC under grant EP/V002325/2.

References

[1] B. Day, R. Street, *Monoidal bicategories and Hopf algebroids*, Advances in Mathematics, Vol. 129, Issue 1, pp. 99-157, (1997)

[2] M. Fiore, N. Gambino, M. Hyland, Monoidal Bicategories, Differential Linear Logic, and Analytic Functors, *In preparation*

[3] S. Lack, *A coherent approach to pseudomonads*, Advances in Mathematics, Vol. 152, pp. 179-202, (2000)

[4] S. Lack, *Limits for Lax Morphisms*, Applied Categorical Structures, Vol. 13, pp. 189–203 (2005).

[5] F. Marmolejo, *Doctrines whose structure forms a fully faithful adjoint string*, Theory and Applications of Categories, Vol. 3, No. 2, pp. 22–23, (1997).

[6] P. McCrudden, *Opmonoidal monads*, Theory and Applications of Categories, Vol. 10, No.19, pp.469–485, (2002).

[7] A. Miranda, Eilenberg-Moore Bicategories for Opmonoidal Pseudomonads, preprint `https://arxiv.org/abs/2402.11703`.

[8] I. Moerdijk, *Monads on tensor categories*, Journal of Pure and Applied Algebra, Vol. 168, Issues 2–3, pp. 189-208, (2002)

[9] A. J. Power, *Three-dimensional monad theory*, Categories in algebra, geometry and mathematical physics, Vol 431, pp 405-426, (2007)

[10] R. Street, *The formal theory of monads*, Journal of Pure and Applied Algebra, Vol. 2, pp. 149-168, (1978)

[11] M. Zawadowski, *The Formal Theory of Monoidal Monads*, Journal of Pure and Applied Algebra, Vol. 216, pp. 1932-1942, (2012)