The dichotomy between enriched and internal categorical structures

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Abstract.

We revisit the dichotomy between enriched and internal categories in a base category $\mathcal{V}$. By describing these objects as monads internal to certain proarrow equipments, we construct a change-of-base adjunction between the category of enriched $\mathcal{V}$-categories and the category of internal $\mathcal{V}$-categories, provided $\mathcal{V}$ satisfies suitable conditions. This perspective allows us to cast enriched $\mathcal{V}$-categories as internal $\mathcal{V}$-categories whose object-of-objects is discrete, which finds applications in the study of the descent theory for $\mathcal{V}$-functors, as shown in [3, Theorem 9.11].

Motivated by the study of the descent theory of functors between $(T, \mathcal{V})$-categories [1], the goal of [4] is to extend these techniques to the setting of generalized multicategories [2]. Using the above dichotomy as our guiding principle, we study the notion of change-of-base for a notion of lax algebras for monads in a suitable 2-category of proarrow equipments. Given suitable conditions on the category $\mathcal{V}$ and the monad T, we obtain an analogous adjunction between enriched and internal $(T, \mathcal{V})$-categories, which we use to describe effective descent functors of $(T, \mathcal{V})$-categories.

References

[CT03] M. M. Clementino and W. Tholen. Metric, topology and multicategory—a common approach. *J. Pure Appl. Algebra*, 179(1-2):13–47, 2003.

[CS10] G. Cruttwell and M. Shulman. A unified framework for generalized multicategories. *Theory Appl. Categ.*, 24(21):580–655, 2010.

[Luc18] F. Lucatelli Nunes. Pseudo-Kan extensions and descent theory. *Theory Appl. Categ.* **33**(15):390–444, 2018.

[PL23] R. Prezado and F. Lucatelli Nunes. Generalized multicategories: change-of-base, embedding and descent. Preprint 23-29, DMUC (2023).