Modules over invertible 1-cocycles

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Abstract.

Hopf braces are recent mathematical objects introduced by I. Angiono, C. Galindo and L. Vendramin in [1] and obtained through a linearisation process from skew braces, which give rise to non-degenerate, bijective and not necessarily involutive solutions of the Quantum Yang-Baxter Equation (see [5]), whose formulation is the following:

$$(\tau \otimes id_V) \circ (id_V \otimes \tau) \circ (\tau \otimes id_V) = (id_V \otimes \tau) \circ (\tau \otimes id_V) \circ (id_V \otimes \tau), \tag{QYBE}$$

where $\tau \colon V \otimes V \to V \otimes V$ is a linear map and V, a $\mathbb{K}$-vector space. As was proven in [1, Corollary 2.4], cocommutative Hopf braces are also relevant from a physical standpoint because they also induce solutions of the above-mentioned equation.

On the one hand, a well-known result for Hopf braces is their strong relationship with invertible 1-cocycles due to the fact that both categories are equivalent (see [1, Theorem 1.12] and [4, Theorem 3.2]). These objects are no more than coalgebra isomorphism between Hopf algebras $\pi \colon H \to B$, related to each other through a module-algebra structure, and satisfying a weaker condition than being algebra morphism.

On the other hand, R. González Rodríguez in [3] introduced the notions of module over a Hopf brace and Hopf module over a Hopf brace, obtaining a categorical equivalence between the base braided monoidal category C and the category of Hopf modules over a Hopf brace, also known by the Fundamental Theorem of Hopf modules for Hopf braces.

Therefore, considering the aforementioned precedents, the aim of this talk is going to be giving a suitable notion of module over a invertible 1-cocycle in such a way that the categorical equivalence between Hopf braces and invertible 1-cocycles remains valid between their module categories.

References

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