Categories of modules, comodules and contramodules over representations

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Abstract.

In [2], Estrada and Virili considered a representation (a functor) $\mathcal{A} : \mathscr{C} \longrightarrow Add$ of a small category $\mathscr{C}$ taking values in Add of small preadditive categories and introduced a concept of modules over such a representation. A sheaf of $\mathcal{O}_X$-modules over a scheme $(X, \mathcal{O}_X)$ is the prototypical example of a module over such a representation.

In algebraic geometry, the idea of studying schemes by means of module categories linked with adjoint pairs given by extension and restriction of scalars is well developed in the literature. In this talk, we will consider representations of a small category taking values in (co)algebras and build an algebraic geometry like categorical framework that studies modules, comodules and contramodules over such representations using adjoint functors. We will discuss the cartesian objects in each of these categories, which may be viewed as counterparts of quasi-coherent sheaves over a scheme. We will focus on understanding the generators for these categories and the Grothendieck categories appearing in these contexts, because the latter may be treated as replacements for noncommutative spaces. This is a recent joint work [1] with Balodi and Banerjee.

References

[1] M. Balodi, A. Banerjee, S. Ray, *Categories of modules, comodules and contramodules over representations*, Forum Mathematicum, Vol.**36**, No.1, 111–152 (2023).

[2] S. Estrada, S. Virili, *Cartesian modules over representations of small categories*, Advances in Mathematics, Vol.**310**, 557–609 (2017).