Homotopy Bicategories of Complete 2-fold Segal Spaces

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Abstract.

Across the multitude of definitions for a higher category, a dividing line can be found between two major camps of model. On one side lives the 'algebraic' models, like Bénabou's bicategories, tricategories following Gurski and the models of Batanin and Leinster [6, ch. 9], Trimble [3] and Penon [9]. These models specify composition operations and higher coherence morphisms, such as associators, all satisfying certain coherence conditions, such as the pentagon condition for bicategories. On the other end, one finds the 'non-algebraic' models, which do not make such specifications and may instead allow many choices of composite. These include the models of Tamsamani [11] and Paoli [8], along with quasicategories [4], complete n-fold Segal spaces [1] and more.

The bridges between these models remain somewhat mysterious. Progress has been made in certain instances, as seen in the work of Tamsamani [11], Lack and Paoli [5], Campbell [2], Moser [7] and others. Nonetheless, the correspondence remains incomplete; indeed, for instance, there is no fully verified means in the literature to take a weak 'algebraic' homotopy n-category of any known model of weak (∞, n)-category for general n.

In this talk, I will present a concrete means to explicitly construct homotopy bicategories of non-algebraic $(\infty, 2)$-categories, in particular Reedy fibrant complete 2-fold Segal spaces. The method, developed in [10], relies on choosing solutions to certain lifting problems, which determine choices of horizontal composition operations. Homotopies between such solutions induce associators and unitors, while coherence conditions are finally induced by higher homotopies between these homotopies. The methods presented extend neatly to obtaining pseudofunctors from maps between complete 2-fold Segal spaces as well. I will compare this construction to other ways one may obtain homotopy bicategories of complete 2-fold Segal spaces elsewhere in the literature, including methods induced by the work of Campbell [2] and of Moser [7].

References

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