Enriched Grothendieck topologies under change of base.

A. Rosenfield

Ariel Rosenfield (rosenfia@uci.edu)
University of California, Irvine

Abstract.
In the presence of a monoidal adjunction

$$F \dashv G : \mathcal{U} \leftrightarrows \mathcal{V}$$

between locally finitely presentable Bénabou cosmoi, we examine the behavior of $\mathcal{V}$-Grothendieck topologies on a $\mathcal{V}$-category $\mathcal{C}$, and that of their constituent covering sieves, under the change of enriching category

$$G_* : \mathcal{V}\text{-Cat} \to \mathcal{U}\text{-Cat}$$

induced by G. In particular, we prove some basic lattice-theoretic properties of the collection of $\mathcal{V}$-Grothendieck topologies on $\mathcal{C}$, and that when G is faithful and conservative, any $\mathcal{V}$-Grothendieck topology on $\mathcal{C}$ corresponds uniquely to a $\mathcal{U}$-Grothendieck topology on $G_*\mathcal{C}$.

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