Limits and colimits in (∞, n)-category theory

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Abstract. The notion of limit for a diagram valued in an ordinary category is a very fundamental one and allows one to encode the universal property of a variety of constructions of interest. This viewpoint becomes even more relevant in the context of higher category theory, where explicit constructions are often challenging to describe, given the infinite amount of coherence involved. For this reason, it becomes crucial to have at one's disposal a meaningful and well-behaved notion for the (possibly weighted) limit of a diagram valued in an n-category or an (∞, n)-category. We will first discuss the universal property for (∞, n)-limits that naturally arises from enriched category theory (in the sense of Borceux-Kelly and Shulman), and mention some of the difficulties which arise in this particular context. We will then propose an alternative formulation, and phrase the universal property for the limit of a diagram valued in an (∞, n)-category in terms of an appropriate double $(\infty, n-1)$-category of cones over such diagram, generalizing at once the viewpoint taken for 2-limits by Grandis-Paré, for n-limits by Moser-Sarazola-Verdugo, and for $(\infty, 1)$-limits by Joyal and Lurie. This is joint work with Moser and Rasekh.

References

[1] L. Moser, N. Rasekh, M. Rovelli, (∞, n)-Limits I: definition and first consistency results, preprint arXiv:2312.11101, 2023.