Formal category theory via ∞-categorical proarrow equipments

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Abstract. The language of Joyal and Lurie's ∞-categories has become an indispensable tool in homotopy theory nowadays. However, some homotopical constructions are better phrased using other flavors of ∞-categories. For instance, it turns out that it is more convenient to work with ∞-categories internal to an ∞-topos of equivariant spaces in the field of equivariant homotopy, while it is essential to work with enriched ∞-categories in other contexts. It would thus be useful to have an overarching framework that produces theories of these types of generalized ∞-categories.

In this talk, we present an extension of methods from *formal* or *synthetic category theory* to the realm of ∞-categories. The field of formal category theory aims to distill the concepts enabling the formulation of well-behaved category theories internal to an ambient 2-category, with the prototypical example being the 2-category of categories. Building upon the foundational work of Street and Walters [4], Wood [1] developed a notion of *proarrow equipments.* This is an axiomatization of structure on a 2-category that allows one to define a good internal notion of pointwise Kan extensions, for instance.

By adopting Shulman's [3] and Verity's [2] double categorical perspective on these equipments, we will see that the theory of equipments naturally extends to the ∞-categorical context. An ∞-categorical equipment gives rise to well-behaved categorical concepts in its underlying $(\infty, 2)$-category such as Kan extensions, exact squares, and - under good conditions - notions of fibrations and comprehensive factorizations. There exist suitable equipments that yield the theory of indexed ∞-categories, more generally, ∞-categories internal to an ∞-topos, enriched ∞-categories, fibered ∞-categories and variants of these. Even more general, equipments that produce internal (∞, n)-category theories, may be constructed. We would like to highlight a few of these examples.

References

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