A 2-categorical model of oriented 1-cobordisms.

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Abstract.

The compact closed quasi category freely generated by one object is a much bigger object than the compact closed category generated by one object, in fact the latter is a mere 1-truncation of the former. Recall that the ∞-category of oriented 1-cobordisms $\mathbf{Bord}_1$ is the compact closed quasi-category freely generated by one object [AF21][H18][Lur09]. Our goal is to construct a symmetric monoidal bicategorical model of $\mathbf{Bord}_1$. Recall that the infinite loop space freely generated by a point $\Omega^\infty\Sigma^\infty(S^0)$ is the classifying space of a symmetric monoidal category Q constructed by Quillen in his proof of the Barrat-Priddy-Quillen theorem [Gra76]. We show that cospans in Q are naturally the 1-cells of a symmetric monoidal bicategory $CosQ$ [Sta16]. The bicategory $CosQ$ can be enlarged with the addition of 2-cells, called creation and a destruction operators. We conjecture that the resulting symmetric monoidal bicategory CCC is a model of the ∞-category $\mathbf{Bord}_1$ (in particular, $\mathbf{Bord}_1(A_0, A_1) \simeq \mathrm{B}CCC(A_0, A_1)$ for any pair of finite signed sets (A_0, A_1)). In support of the conjecture, we show that $CCC(\emptyset, \emptyset)$ is the symmetric monoidal category freely generated by Connes's cyclic category Λ and the space $\mathbf{Bord}_1(\emptyset, \emptyset)$ is the E_∞-space freely generated by $\mathbb{C}P^\infty$ (Recall that $\mathrm{B}\Lambda = \mathbb{C}P^\infty$ [DHK85]).

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