Lax adjunctions and lax-idempotent pseudomonads

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Abstract.

Just as a family of U-universal maps $\eta_A : A \to UFA$ for a functor U gives rise to an adjunction, a family of maps satisfying certain "relative U-left Kan extension" conditions for a pseudofunctor U gives rise to a **lax adjunction**. We begin by presenting a generalization of Marta Bunge's result [1, Theorem 4.1] where this has been proven for 2-functors.

We then apply this result to the setting of **lax-idempotent pseudomonads** where we introduce a new technique for creating lax adjunctions out of biadjunctions. We give various examples, for instance we show that there is a canonical lax adjunction between the 2-category of algebras and the Kleisli 2-category for such pseudomonads. Another application guarantees a certain *enriched weak completeness* (in the sense of [3]) of the Kleisli 2-category. This for example applies to the bicategory PROF of locally small categories and "small" profunctors.

Finally, we show how (the dual of) this technique provides us with lax versions of classical results from two-dimensional monad theory, as described in [2, Section 5]. For instance, the 2-category of monoidal categories and lax monoidal functors, while not having many bicolimits, is weakly cocomplete in the above sense.

References

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[3] S. Lack, J. Rosický, *Enriched weakness*, Journal of Pure and Applied Algebra, 216.8-9 (2012): 1807-1822.