A classifying localic category for locally compact locales

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Abstract. It is reasonably clear how to find a classifying localic groupoid for locally compact locales. That is, we can find a localic groupoid $\mathbb{G}$ such that principal $\mathbb{G}$-bundles over any locale X are equivalent to locally compact locales in the topos of sheaves, $Sh(X)$. This is like a localic version of having a classifying topos for geometric theories, but not quite as the morphisms between principal bundles are all isomorphisms. So really, all we are 'classifying' are locally compact locales with isomorphisms between them, not locale maps in general. This is in contrast to classifying toposes which classify the models of a geometric theory *and* their morphisms. Further it seems hard to avoid this problem since, as is well known, all morphisms between principal bundles are isomorphisms. In this talk I'll present a way round the issue by defining morphisms between principal bundles as certain principal bundles associated with the arrow category naturally arising in the construction of $\mathbb{G}$.

In fact the underlying results are quite general and can be summarised by a nice characterisation of geometric stacks of *categories* on any cartesian category $\mathcal{C}$. Here a geometric stack of *categories* is just a geometric stack of groupoids (i.e. $X \mapsto Prin_{\mathbb{G}}(X)$) but using the new notion of morphism between principal bundles to define the morphisms of each category of principal bundles $Prin_{\mathbb{G}}(X)$.