Associativity of Cosmash Products in Non-associative Algebras

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Abstract.

The purpose of this talk is to convince you that, for a field K with characteristic zero, the condition called *cosmash associativity* characterizes the variety of commutative and associative K-algebras amongst all varieties of non-associative K-algebras. Since this condition is motivated by Category Theory, this gives a very abstract way of thinking about commutativity and associativity.

In order to present things in an understandable way, we will first recall the notion of the *binary cosmash product*. We see how it naturally leads to a suitable definition of binary commutators by looking at some classical examples. In the case of commutative associative algebras, it corresponds to the binary tensor product. We then try to extend these notions to the ternary case, and even to the n-ary case for some natural number n. From this point, what *cosmash associativity* means can be explained essentially without effort. In the end, we discuss the main result and the techniques used to prove it. If time allows it, we mention new questions which appeared from this project.

Joint work with Ülo Reimaa and Tim Van der Linden.

References

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