Constructing generalized schemes using cone injectivity

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Abstract.

We provide generalizations of two notions of a scheme from algebraic geometry. Both are defined as certain geometric objects admitting an open cover by "affine schemes". In the first approach, affine schemes constitute the image of a spectrum functor valued in a suitable category generalizing that of locally ringed spaces. The spectrum functor is constructed using cone injectivity and the construction works quite generally, especially for a locally finitely presentable category $\mathcal{A}$. In this full generality however, the spectrum functor fails to be fully faithful and we explain reasonable sufficient conditions under which it is. In the second approach, we develop a generalization of another concept from algebraic geometry – the functor of points, valued in a certain category of (small) sheaves on $\mathcal{A}^{\mathrm{op}}$. Finally, assuming the full faithfulness of the spectrum functor, we prove equivalence of the two resulting notions of schemes. On the way, we prove a useful universal property of the category of small sheaves.

This is based on a joint work with J. Jurka and T. Perutka, written up in an arxiv preprint [1].

References

[1] J. Jurka, T. Perutka, L. Vokřínek, Constructing generalized schemes using cone injectivity, preprint, arXiv:2201.03516v3, 2023.