A monotone-light factorization for n-categories

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Abstract.

Starting with a symmetric monoidal adjunction with certain properties, we derive another symmetric monoidal adjunction with the same properties between the respective categories of all $\mathcal{V}$-categories. If we begin with a reflection of a full replete subcategory, the derived adjunction is also a reflection of the same kind. Semi-left-exactness (also called admissibility in categorical Galois theory) or the stronger stable units property is inherited by the derived reflection. Applying these results, we conclude that the reflection of the category of all n-categories into the category of n-preorders has stable units. Then, it is also shown that this reflection determines a monotone-light factorization system on n-categories, $n \geq 1$, and that the light morphisms are precisely the n-functors faithful with respect to n-cells. In order to achieve such results, it was also shown that n-functors surjective both on vertically composable triples of horizontally composable pairs of n-cells, and on horizontally composable triples of vertically composable pairs of n-cells, are effective descent morphisms in the category of all n-categories $nCat$, $n \geq 1$.

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