Toposes vs Localic Groupoids:
A unified treatment of covering theorems

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Abstract. The idea of toposes as spaces whose points have non-trivial automorphisms is said to have originated with Grothendieck, and was first made concrete by Joyal–Tierney [4]. They showed that every Grothendieck topos $\mathcal{E}$ admits a localic groupoid $\mathcal{L}(\mathcal{E})$ presenting $\mathcal{E}$, in the sense that the topos of equivariant sheaves on $\mathcal{L}(\mathcal{E})$ is equivalent to $\mathcal{E}$. This idea was then revisited by many authors (Moerdijk [5], Butz–Moerdijk [3], Awodey–Forssell [1]); there are nowadays plenty of variations on this theme in the literature. These covering theorems, or viewed another way, reconstruction theorems depend on different assumptions and are rarely directly comparable, but their proofs turn out to follow a general pattern.

In this talk, we abstract that pattern and thus reduce such a reconstruction theorem to its bare minimum; we call the minimal data associated to such a theorem its *amorphous sheaf*. These are very concrete objects: a locale and a sheaf over it satisfying certain properties. We will explain how to recover a reconstruction theorem from an amorphous sheaf, recalling the necessary amount of descent theory along the way.

Due to their tangible nature, amorphous sheaves can readily be studied from a logical point of view. For instance, we provide a logical recognition criterion for amorphous sheaves; this relies on the theory of classifying toposes for first-order theories of Butz–Johnstone [2]. Finally, we illustrate the resulting framework by reviewing a selection of established reconstruction theorems.

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References

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